# Problem Solutions

## Chapter Seven: Field-Effect Transistors

P7.1. Calculate the zero-bias threshold voltage for an n-channel MOSFET with $t_{ox} = 6\text{nm}$ and $N_a = 1.2\times10^{16}\,cm^{-3}$. Assume that the gate is heavily-doped n-polysilicon (with the Fermi level coincident with the conduction band) and that there are $10^{11}\,cm^{-2}$ positive charges in the oxide. A boron dose of $10^{12}\,cm^{-2}$ is implanted to adjust the threshold voltage.

**Solution.** The zero bias threshold is

$$V_{TO} = \phi_{MS} - 2\phi_F - \frac{Q_B}{C_{ox}} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{II}}{C_{ox}}.$$

For this level of doping,

$$\phi_F = \frac{kT}{q}\ln\left(\frac{n_i}{N_a}\right) = (0.026V)\ln\left(\frac{1.45\times10^{10}\,cm^{-3}}{1.2\times10^{16}\,cm^{-3}}\right) = -0.35V.$$

The work function difference is

$$\phi_{MS} = \frac{kT}{q}\ln\left(\frac{n_i}{N_a}\right) - \frac{E_g}{2q} = -0.35V - \frac{1.12V}{2} = -0.90V.$$

The contribution due to the depletion charge in the semiconductor under inversion is

$$\frac{Q_B}{C_{ox}} = \frac{-\sqrt{2qN_a\varepsilon_{Si}|2\phi_F|}}{\varepsilon_{ox}/t_{ox}}$$
$$= \frac{-\sqrt{2(1.602\times10^{-19}C)(1.2\times10^{16}cm^{-3})(11.9)(8.85\times10^{-14}F/cm)|2(-0.35V)|}}{(3.9)(8.85\times10^{-14}F/cm)/6\times10^{-7}cm}$$
$$= -0.092V$$

The contribution due to the oxide charge is

$$\frac{Q_{ox}}{C_{ox}} = \frac{\left(1.602 \times 10^{-19}\,C\right)\left(10^{11}\,cm^{-2}\right)}{\left(3.9\right)\left(8.85 \times 10^{-14}\,F/cm\right)/6 \times 10^{-7}\,cm} = 0.028V.$$

The adjustment due to the implantation of boron is

$$\frac{Q_{II}}{C_{ox}} = -\frac{\left(1.602 \times 10^{-19}\,C\right)\left(10^{12}\,cm^{-2}\right)}{\left(3.9\right)\left(8.85 \times 10^{-14}\,F/cm\right)/6 \times 10^{-7}\,cm} = -0.28V.$$

Therefore the zero-bias threshold voltage is

$$\begin{aligned} V_{TO} &= \phi_{MS} - 2\phi_F - \frac{Q_B}{C_{ox}} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{II}}{C_{ox}} \\ &= -0.90V + 0.70V + 0.092V - 0.028V + 0.28V. \\ &= 0.144V \end{aligned}$$

P7.2. Calculate the zero-bias threshold voltage for a p-channel MOSFET with $t_{ox} = 6\text{nm}$ and $N_d = 10^{16}\,cm^{-3}$. Assume that the gate is heavily-doped p-polysilicon (with the Fermi level coincident with the valence band) and that there are $10^{11}\,cm^{-2}$ positive charges in the oxide. A phophorus dose of $5 \times 10^{11}\,cm^{-2}$ is implanted to adjust the threshold voltage.

**Solution.** The zero bias threshold is

$$V_{TO} = \phi_{MS} - 2\phi_F - \frac{Q_B}{C_{ox}} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{II}}{C_{ox}}.$$

For this level of doping,

$$\phi_F = -\frac{kT}{q}\ln\left(\frac{n_i}{N_d}\right) = \left(0.026V\right)\ln\left(\frac{1.45 \times 10^{10}\,cm^{-3}}{10^{16}\,cm^{-3}}\right) = 0.35V.$$

The work function difference is

$$\phi_{MS} = -\frac{kT}{q}\ln\left(\frac{n_i}{N_a}\right) + \frac{E_g}{2q} = 0.35V + \frac{1.12V}{2} = 0.90V.$$

The contribution due to the depletion charge in the semiconductor under inversion is

$$\frac{Q_B}{C_{ox}} = \frac{\sqrt{2qN_d\varepsilon_{Si}|2\phi_F|}}{\varepsilon_{ox}/t_{ox}}$$

$$= \frac{\sqrt{2(1.602\times10^{-19}C)(10^{16}cm^{-3})(11.9)(8.85\times10^{-14}F/cm)2(0.35V)}}{(3.9)(8.85\times10^{-14}F/cm)/6\times10^{-7}cm}$$

$$= 0.084V$$

The contribution due to the oxide charge is

$$\frac{Q_{ox}}{C_{ox}} = \frac{(1.602\times10^{-19}C)(10^{11}cm^{-2})}{(3.9)(8.85\times10^{-14}F/cm)/6\times10^{-7}cm} = 0.028V.$$

The adjustment due to the implantation of phosphorus is

$$\frac{Q_{II}}{C_{ox}} = \frac{(1.602\times10^{-19}C)(5\times10^{11}cm^{-2})}{(3.9)(8.85\times10^{-14}F/cm)/6\times10^{-7}cm} = 0.139V.$$

Therefore the zero-bias threshold voltage is

$$V_{TO} = \phi_{MS} - 2\phi_F - \frac{Q_B}{C_{ox}} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{II}}{C_{ox}}$$

$$= 0.90V - 0.70V - 0.084V - 0.028V - 0.139V.$$

$$= -0.051V$$

P7.3. Consider an n-channel MOSFET with $t_{ox} = 7\text{nm}$ and $N_a = 10^{16}cm^{-3}$. Determine the required ion implantation (impurity and dose) to adjust the threshold by +0.45V.

**Solution.** The ion implantation shifts the threshold voltage by

$$\Delta V_{TO} = -\frac{Q_{II}}{C_{ox}}.$$

A positive shift in the threshold requires the implantation of negative charge (Boron). The required ion implantation charge per unit area is

$$\begin{aligned} Q_{II} / A &= -\Delta V_{TO} C_{ox} / A \\ &= -\Delta V_{TO} \varepsilon_{ox} / t_{ox} \\ &= -(0.45V)(3.9)(8.85 \times 10^{-14} F/cm)/(7 \times 10^{-7} cm) \\ &= -2.2 \times 10^{-7} C/cm^2 \end{aligned}$$

The required dose of Boron is

$$N_{II} = \frac{Q_{II} / A}{-q} = \frac{-2.2 \times 10^{-7} C/cm^2}{-1.602 \times 10^{-19} C} = 1.37 \times 10^{12} cm^{-2} .$$

P7.4. Consider an n-channel MOSFET with $t_{ox} = 7\text{nm}$ and $N_a = 10^{16} cm^{-3}$. Assume that the gate is heavily-doped n-polysilicon (with the Fermi level coincident with the conduction band) and that there are $10^{11} cm^{-2}$ positive charges in the oxide. Determine the required ion implantation (impurity and dose) to adjust the threshold voltage to 0.4V.

**Solution.** The required shift in the threshold is

$$\Delta V_{TO} = V_{TO} - \phi_{MS} + 2\phi_F + \frac{Q_B}{C_{ox}} + \frac{Q_{ox}}{C_{ox}} .$$

For this level of doping,

$$\phi_F = \frac{kT}{q} \ln\left(\frac{n_i}{N_a}\right) = (0.026V) \ln\left(\frac{1.45 \times 10^{10} cm^{-3}}{10^{16} cm^{-3}}\right) = -0.35V .$$

The work function difference is

$$\phi_{MS} = \frac{kT}{q} \ln\left(\frac{n_i}{N_a}\right) - \frac{E_g}{2q} = -0.35V - \frac{1.12V}{2} = -0.90V .$$

The contribution due to the depletion charge in the semiconductor under inversion is

$$\frac{Q_B}{C_{ox}} = \frac{-\sqrt{2qN_a\varepsilon_{Si}|2\phi_F|}}{\varepsilon_{ox}/t_{ox}}$$

$$= \frac{-\sqrt{2(1.602\times10^{-19}C)(10^{16}cm^{-3})(11.9)(8.85\times10^{-14}F/cm)2(-0.35V)|}}{(3.9)(8.85\times10^{-14}F/cm)/7\times10^{-7}cm}$$

$$= -0.099V$$

The contribution due to the oxide charge is

$$\frac{Q_{ox}}{C_{ox}} = \frac{(1.602\times10^{-19}C)(10^{11}cm^{-2})}{(3.9)(8.85\times10^{-14}F/cm)/7\times10^{-7}cm} = 0.032V\ .$$

Therefore the required shift in the threshold is

$$\Delta V_{TO} = V_{TO} - \phi_{MS} + 2\phi_F + \frac{Q_B}{C_{ox}} + \frac{Q_{ox}}{C_{ox}}$$

$$= 0.4V + 0.90V - 0.70V - 0.099V + 0.032V\ .$$

$$= 0.53V$$

Because the result is positive, the required impurity is Boron. The required dose is

$$N_{II} = \frac{\Delta V_{TO}C_{ox}/A}{q} = \frac{(0.53V)(3.9)(8.85\times10^{-14}F/cm)/7\times10^{-7}cm}{1.602\times10^{-19}C} = 1.63\times10^{12}cm^{-2}\ .$$

P7.5. Consider n-channel and p-channel silicon MOSFETs fabricated on the same wafer with channel lengths of 0.5 μm and 20 nm thick silicon dioxide.

a. Determine the process transconductance parameters for n-channel and p-channel devices.

b. Determine the required aspect ratios for n-channel and p-channel MOSFETs such that the device transconductance parameters are both 0.5 mA/V$^2$.

c. Determine the oxide capacitances of the devices. (The oxide capacitance is approximately the parallel plate capacitance for plates of area $W \times L$ and separation $t_{ox}$.)

**Solution.**

a. The process transconductance parameters are

$$k'_P = \frac{(230cm^2/Vs)(3.9)(8.85\times10^{-14}F/cm)}{20\times10^{-7}cm} = 0.040mA/V^2$$

and

$$k'_N = \frac{(580cm^2/Vs)(3.9)(8.85\times10^{-14}F/cm)}{20\times10^{-7}cm} = 0.100mA/V^2.$$

b. The required aspect ratios are

$$\frac{W_P}{L_P} = \frac{0.5mA/V^2}{0.040mA/V^2} = 12.5$$

and

$$\frac{W_N}{L_N} = \frac{0.5mA/V^2}{0.100mA/V^2} = 5.$$

c. The oxide capacitances are

$$C_{oxP} = \frac{(3.9)(8.85\times10^{-14}F/cm)(0.5\times10^{-4}cm)(6.25\times10^{-4})}{20\times10^{-7}cm} = 5.4fF$$

and

$$C_{oxN} = \frac{(3.9)(8.85\times10^{-14}F/cm)(0.5\times10^{-4}cm)(2.5\times10^{-4})}{20\times10^{-7}cm} = 2.2fF.$$

P7.6. Consider n-channel and p-channel silicon MOSFETs fabricated on the same wafer with channel lengths of 0.3 μm and 14 nm thick silicon dioxide.

a. Determine the process transconductance parameters for n-channel and p-channel devices.

b. Determine the required aspect ratios for n-channel and p-channel MOSFETs such that the device transconductance parameters are both 0.5 mA/V$^2$.

c. Determine the oxide capacitances of the devices. (The oxide capacitance is approximately the parallel plate capacitance for plates of area $W \times L$ and separation $t_{ox}$.)

**Solution.**

a. The process transconductance parameters are

$$k_P' = \frac{(230cm^2/Vs)(3.9)(8.85\times10^{-14}F/cm)}{14\times10^{-7}cm} = 0.056mA/V^2 \text{ and}$$

$$k_N' = \frac{(580cm^2/Vs)(3.9)(8.85\times10^{-14}F/cm)}{14\times10^{-7}cm} = 0.142mA/V^2 .$$

b. The required aspect ratios are

$$\frac{W_P}{L_P} = \frac{0.5mA/V^2}{0.056mA/V^2} = 8.9 \text{ and}$$

$$\frac{W_N}{L_N} = \frac{0.5mA/V^2}{0.142mA/V^2} = 3.5 .$$

c. The oxide capacitances of the devices are

$$C_{OXP} = \frac{(3.9)(8.85\times10^{-14}F/cm)(0.3\times10^{-4}cm)^2(8.9)}{14\times10^{-7}cm} = 2.0fF \text{ and}$$

$$C_{OXN} = \frac{(3.9)(8.85\times10^{-14}F/cm)(0.3\times10^{-4}cm)^2(3.5)}{14\times10^{-7}cm} = 0.78fF .$$

P7.7. Calculate and plot the characteristic curves ($I_D$ versus $V_{DS}$ with $V_{GS}$ as a parameter) for an n-channel MOSFET with $W = 2\mu m$, $L = 0.25\mu m$, $t_{ox}$ = 6nm and $V_T = 0.4V$. Consider $0 \le V_{DS} \le 2.5V$, and $V_{GS}$ = 0, 0.5V, 1V, 1.5V, 2V, and 2.5V.

P7.8. Calculate and plot the subthreshold current for the following n-channel MOSFETs a) a bulk n-channel MOSFET with $K = 1.6mA/V^2$, $V_T = 0.3V$, and $m = 1.6$.

b) a silicon-on-insulator (SOI) n-channel MOSFET with $K = 1.6mA/V^2$, $V_T = 0.3V$, and $m = 1$.

P7.9. The *destructive* breakdown of silicon dioxide occurs with an electric field of about 10 million volts per centimeter. Consider an integrated MOSFET with a 4.5 nm thick oxide.

a. Determine the gate-to-source bias voltage which will result in destructive breakdown.

b. Determine the associated charge in Coulombs for a $0.18\mu m \times 2\mu m$ gate.

c. Determine the associated number of electrons.

**Solution.**

a. The gate to source bias voltage which will result in destructive breakdown is

$$|V_{GS}| = \left(10^7 V/cm\right)\left(45 \times 10^{-8} cm\right) = 4.5V .$$

b. The associated charge in Coulombs for a $0.18\mu m \times 2\mu m$ gate is

$$Q = CV = \frac{(3.9)\left(8.85 \times 10^{-14} F/cm\right)\left(0.18 \times 10^{-4} cm\right)\left(2 \times 10^{-4}\right)(4.5V)}{45 \times 10^{-8} cm} = 12.4 fC$$

c. The associated number of electrons is

$$N = \frac{12.4 \times 10^{-15} C}{1.602 \times 10^{-19} C/electron} = 78{,}000\ electrons .$$

P7.10. Estimate the transit time for an n-channel MOSFET with $L = 100nm$ and $V_{DS} = 1.5V$, assuming

a. the constant mobility model, and

b. the velocity saturation model.

Which model is more appropriate in this case?

**Solution.**

a. Assuming a constant electron mobility of 580 $cm^2/Vs$, the transit time is

$$t_t = \frac{L^2}{\mu_n V_{DS}} = \frac{\left(100\times10^{-7}\,cm\right)^2}{\left(580cm^2V^{-1}s^{-1}\right)\left(1.5V\right)} = 0.115ps\,.$$

b. Assuming a saturated velocity of $10^7$ cm/s, the transit time is

$$t_t = \frac{L}{v_{sat}} = \frac{100\times10^{-7}\,cm}{10^7\,cm/s} = 1.0ps\,.$$

The saturated velocity model is appropriate and the constant mobility model underestimates the transit time.