1.

(a) $5\sin\left(5t - \frac{9\pi}{180}\right),\ t = 0, 0.01, 0.1\text{ s}:$

-0.78; -0.53; 1.7

(b) $4\cos 2t,\ t = 0, 1, 1.5\text{ s}:$

4.0, -1.7, -4.0

(c) $3.2\cos\left(6t + \frac{15\pi}{180}\right),\ t = 0, 0.01, 0.1:$

3.1, 3.0, 2.1

2. (a)

$$300\sin 628t = \boxed{300\cos(628t - 90°);}$$

$$4\sin(3\pi t + 30°) = 4\cos(3\pi t + 30° - 90°) = \boxed{4\cos(3\pi t - 60°);}$$

$$14\sin(50t - 5°) - 10\cos 50t = -10\cos 50t + 14\left[\cos(-5°)\sin 50t - \sin(-5°)\cos 50t\right]$$
$$= -10\cos 50t + 13.95\sin 50t + 1.22\cos 50t = -8.78\cos 50t + 13.95\sin 50t$$
$$= \sqrt{(8.78)^2 + (13.95)^2}\cos\left(50t + \tan^{-1}\frac{13.95}{8.78}\right) = \boxed{16.48\cos(50t + 57.8°)}$$

(b)

$$2\cos(100t + 45°) = 2\sin(100t + 45° + 90°) = \boxed{2\sin(100t + 135°);}$$

$$3\cos 4000t = \boxed{3\sin(4000t + 90°);}$$

$$5\cos(2t - 90°) + 10\sin(2t) = 5\cos(2t - 90°) + 10\cos(2t - 90°) = \boxed{15\cos(2t - 90°)}$$

3.

$v_1 = 10\cos(10t - 45°)$

$(a)\ i_1 = 5\cos 10t \quad \therefore \quad \boxed{v_1 \text{ leads } i_1 \text{by } -45°}$

$(b)\ i_1 = 5\cos\left(10t - 80°\right) \quad \therefore \quad \boxed{v_1 \text{ leads } i_1 \text{by } -45-(-80) = +35°}$

$(c)\ i_1 = 5\cos\left(10t - 40°\right) \quad \therefore \quad \boxed{v_1 \text{ leads } i_1 \text{by } -45-(-40) = -5°}$

$(d)\ i_1 = 5\cos\left(10t + 40°\right) \quad \therefore \quad \boxed{v_1 \text{ leads } i_1 \text{by } -45-40 = -85°}$

$(e)\ i_1 = 5\cos\left(10t - 19°\right) \quad \therefore \quad \boxed{v_1 \text{ leads } i_1 \text{by } -45-(-19) = -26°}$

4.

$v_1 = 3\cos(10^4 t - 5°)$ lags

$(a)\ 5\cos 10^4 t$ by $0-(-5) = \boxed{+5°}$

$(b)\ 5\cos\left(10^4 t - 14°\right)$ by $-14-(-5) = \boxed{-9°}$

$(c)\ 5\cos\left(10^4 t - 23°\right)$ by $-23-(-5) = \boxed{-18°}$

$(d)\ 5\cos\left(10^4 t + 23°\right)$ by $23-(-5) = \boxed{28°}$

$(e)\ \sin\left(10^4 t - 390°\right) = 5\cos\left(10^4 t - 30° - 90°\right)$ by $-120-(-5) = \boxed{-115°}$

5. (a) $\cos 4t = \sin(4t + 90°)$ so $\boxed{\sin 4t \text{ lags } \cos 4t}$ (by 90°)

(b) $\boxed{\cos\left(4t - 80°\right) \text{ lags } \cos 4t}$ (by 80°)

(c) $\boxed{\cos 4t \text{ lags } \cos(4t + 80°)}$ (by 80°)

(d)

$$-5\sin 5t = 5\sin(5t + 180°) = 5\cos(5t + 90°)$$

so

$\boxed{\cos(5t + 2°) \text{ lags } -5\sin 5t}$ (by $90 - 2 = 88°$)

(e)

$$\sin 5t + \cos 5t = \sqrt{2}\cos(5t + \tan^{-1}(-1)) = \sqrt{2}\cos(5t - 45^\circ)$$

which $\boxed{\text{neither leads nor lags}}$ $\cos(5t - 45^\circ)$!

6. (a)

$$\cos 3t - 7\sin 3t = \sqrt{1+49}\cos(3t + \tan^{-1} 7) = 7.07\cos(3t + 81.9°)$$

$$= 0 \text{ at } 3t + 81.9° = 3t + \frac{81.9\pi}{180} = \frac{\pi}{2}n,\ n = 1,3,5,...$$

or $t = 0.0471$ s, 1.09 s, and 6.42 s

(b) already a single sinusoid so zero at $10t + 45° = 10t + \frac{45\pi}{180} = \frac{\pi}{2}n,\ n = 1,3,5,...$

or $t = 0.0785$ s, 0.393 s, and 0.707 s

(c)

$$\cos 5t - \sin 5t = \sqrt{1+1}\cos(5t + \tan^{-1}(1)) = \sqrt{2}\cos(5t + 45°)$$

$$= 0 \text{ at } 5t + \frac{45\pi}{180} = \frac{\pi}{2}n,\ n = 1,3,5$$

or $t = 0.157$ s, 0.785 s, and 1.41 s

7. (a)

$$\cos 3t - 7\sin 3t = \sqrt{1+49}\cos(3t + \tan^{-1}(7)) = \sqrt{50}\cos(3t + 81.9°)$$

$= 1$ at $\cos(3t + 81.9°) = 0.1414$

or

$$3t + \frac{81.9\pi}{180} = 1.429$$

Solving, $t = -1.42\times10^{-4}$ s, which is not a valid solution.

Adding 2π,

$$3t = 1.429 + 2\pi - \frac{81.9\pi}{180}, \text{ or } \boxed{t = 2.094 \text{ s}}$$

Adding 4π,

$$3t = 1.429 + 4\pi - \frac{81.9\pi}{180}, \text{ or } \boxed{t = 4.189 \text{ s}}$$

$$\cos(10t + 45°) = 1$$

or

$$10t = \cos^{-1}(1) - 45°$$

Solving, $t = -0.0785$ s, which is not a valid solution.

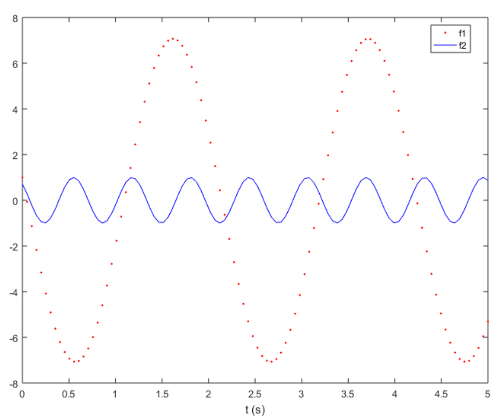
Adding 2π,

$$10t + 45° = 0 + 2\pi, \text{ or } \boxed{t = 0.5498 \text{ s}}$$

Adding 4π,

$$\boxed{t = 1.178 \text{ s}}$$

(b)




```
>> t = linspace(0,5,100);
>> f1 = cos(3*t)-7*sin(3*t);
>> f2 = cos(10*t+45*pi/180);
>> plot(t,f1,'r.',t,f2,'b-');legend;xlabel('t (s)');
>>
```

8. (a) (0,0); (0.5,1): $v-0=\dfrac{1-0}{0.5-0}(t-0)$ or $v(t)=2t$

Thus, $v(0.25) = 2(0.25) = \boxed{0.50 \text{ V}}$

(b)

$$v(t)\approx\frac{8}{\pi^2}\sin \pi t$$

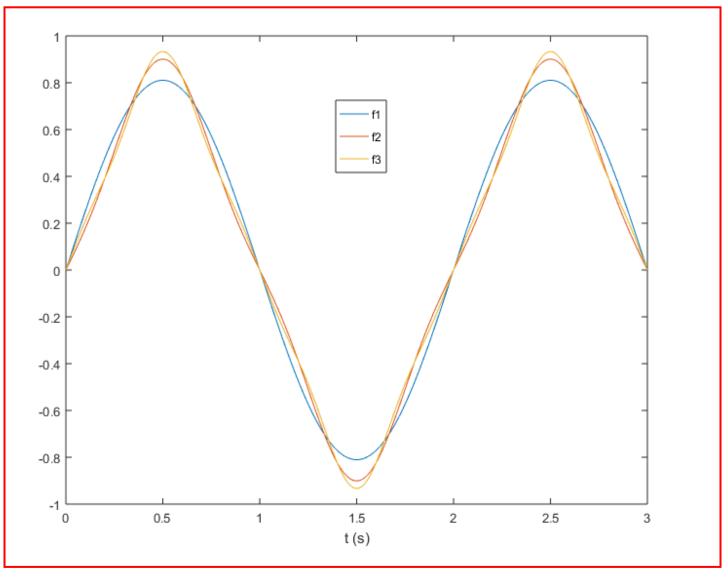
$$so\ v(0.25)=\frac{8}{\pi^2}\sin 0.25\pi=\boxed{0.573 \text{ V}}$$

(c)

$$v(t)\approx\frac{8}{\pi^2}\left(\sin \pi t-\frac{1}{9}\sin 3\pi t+\frac{1}{25}\sin 5\pi t\right)$$

$$so\ v(0.25)=\frac{8}{\pi^2}\left(\sin 0.25\pi-\frac{1}{9}\sin 0.75\pi+\frac{1}{25}\sin 1.25\pi\right)=\boxed{0.487 \text{ V}}$$

(d,e,f)




```
>> t = linspace(0,3,500);
>> f1 = (8/pi^2)*sin(pi*t);
>> f2 = (8/pi^2)*(sin(pi*t)-(1/9)*sin(3*pi*t));
>> f3 = (8/pi^2)*(sin(pi*t)-(1/9)*sin(3*pi*t)+(1/25)*sin(5*pi*t));
>> plot(t,f1,t,f2,t,f3);xlabel('t (s)');
>>
```

9. (a)

$$V_{rms} = \left[\frac{1}{T} \int_0^T V_m^2 \cos^2(\omega t) dt \right]^{\frac{1}{2}}$$

$$= \frac{V_m}{\sqrt{T}} \left[\int_0^T V_m^2 \cos^2(\omega t) dt \right]^{\frac{1}{2}}$$

with $u = \omega t$ and $du = \omega dt$

$$V_{rms} = \frac{V_m}{\sqrt{T}} \left[\int_0^{2\pi} \cos^2 u \; du \right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{T}} \left[\frac{T}{2\pi} \left(\frac{u}{2} + \frac{\sin 2u}{4} \right)_0^{2\pi} \right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{T}} \left[\frac{T}{2\pi} \left(\frac{2\pi}{2} + 0 - 0 - 0 \right) \right]^{\frac{1}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

(proof/derivation)

(b) 110 V_{rm}s corresponds to $110\sqrt{2}$ = **156 V peak**

115 V_{rms} corresponds to $115\sqrt{2}$ = **163 V peak**

120 V_{rms} corresponds to $120\sqrt{2}$ = **170 V peak**

10. Find the Thévenin equivalent by removing the inductor.

(a)

$$v_{TH} = \frac{1}{11} v_s = \frac{4.53}{11}\cos(30t)\ \text{V}$$

$$\text{R}_{TH} = 10 \,||\, 1 = \frac{10}{11}\ \Omega$$

From Eq.[4],

$$i_L(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$= \frac{\left(4.53/11\right)}{\sqrt{\left(\frac{10}{11}\right)^2 + 0.0081}}\cos\left(30t - \tan^{-1}\frac{(30)(0.003)}{10/11}\right)$$

$$= 0.451\cos(30t - 5.65°)\ \text{A}$$

So, $i_L(0) = 0.451\cos(-5.65°) =$ $\boxed{0.449\ \text{A}}$

(b)

$$v_L(t) = L\frac{di_L}{dt} = -(0.0003)(0.451)(30)\sin(30t - 5.65°)$$

$$= -0.0406\sin(30t - 5.65°)\ \text{V}$$

$$= 0.0406\sin(30t + 174.35°)\ \text{V} \quad \text{(if positive magnitude desired)}$$

11. Find the Thévenin equivalent after a quick source transformation. So,

$$V_{oc} = (25\cos 100t)(1)\frac{2}{1+1+2} = 12.5\cos 100t \text{ V and } R_{TH} = 1 \| 1 = 2\ \Omega$$

From Eq. [4],

$$i_L(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$= \frac{12.5}{\sqrt{1+1}}\cos\left(100t - \tan^{-1}\frac{1}{1}\right)$$

$$= \boxed{8.84\cos(100t - 45°) \text{ A}}$$

12. Find the Thévenin equivalent after a quick source transformation. So,

$$V_{oc} = (25\cos 100t)(1)\frac{2}{1+1+2} = 12.5\cos 100t \text{ V} \text{ and } R_{TH} = 1\,||\,1 = 2\ \Omega$$

From Eq. [4],

$$i_L(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$= \frac{12.5}{\sqrt{1+1}}\cos\left(100t - \tan^{-1}\frac{1}{1}\right)$$

$$= 8.84\cos(100t - 45°) \text{ A}$$

$$v_L = L\frac{di_L}{dt} = -10\times 10^{-3}(8.84)(100)\sin(100t-45°)$$

$$= -8.84\sin(100t-45°) = 8.84\sin(100t+135°) \text{ V}$$

This is also the voltage across the 2 Ω resistor, so

$$P_{2\Omega} = \frac{v_L^2}{2} = \frac{1}{2}(8.84)^2\sin^2(100t+135°)$$

$$= \boxed{39.1\sin^2(100t+135°) \text{ W}}$$

13.

$3\cos 4t = v_C + 5i_C$

where

$i_C = C\dfrac{dv_C}{dt}.$

$Thus,\quad 3\cos 4t = v_C + 10\dfrac{dv_C}{dt}$

$With\ v_C(t) = V_1\cos 4t + V_2\sin 4t,$

$\dfrac{dv_C}{dt} = -4V_1\sin 4t + 4V_2\cos 4t$

and so

$V_1\cos 4t + V_2\sin 4t + 40V_2\cos 4t - 40V_1\sin 4t = 3\cos 4t$

We now can obtain two equations in our two unknowns:

$V_1 + 40V_2 = 3 \quad [1]$

$-40V_1 + \quad V_2 = 0 \quad [2]$

Solving,

$V_1 = 0.00187\ \text{V}$

$V_2 = 0.0750\ \text{V}$

Thus,

$$v_C(t) = 0.00187\cos 4t + 0.0750\sin 4t = \sqrt{(0.00187)^2 + (0.0750)^2}\cos\left(4t + \tan^{-1}\frac{(-0.075)}{0.00187}\right)$$

$$= \boxed{0.075\cos(4t - 88.6°)\ \text{V}}$$

14.

$3\cos 4t = v_C + 5i_C$

where

$i_C = C\frac{dv_C}{dt}.$

Thus, $3\cos 4t = v_C + 10\frac{dv_C}{dt}$

With $v_C(t) = V_1\cos 4t + V_2\sin 4t,$

$\frac{dv_C}{dt} = -4V_1\sin 4t + 4V_2\cos 4t$

and so

$V_1\cos 4t + V_2\sin 4t + 40V_2\cos 4t - 40V_1\sin 4t = 3\cos 4t$

We now can obtain two equations in our two unknowns:

$V_1 + 40V_2 = 3 \quad [1]$

$-40V_1 + V_2 = 0 \quad [2]$

Solving,

$V_1 = 0.00187\text{ V}$

$V_2 = 0.0750\text{ V}$

Thus,

$v_C(t) = 0.00187\cos 4t + 0.0750\sin 4t = \sqrt{(0.00187)^2 + (0.0750)^2}\cos\left(4t + \tan^{-1}\frac{(-0.075)}{0.00187}\right)$

$= 0.075\cos(4t - 88.6°)\text{ V}$

Since

$w_C(t) = \frac{1}{2}Cv_C^2 = (\frac{1}{2})(2)\left[0.075\cos(4t - 88.6°)\right]^2$

$= 0.0563\cos^2(4t - 88.6°)\text{ J}$

At $t = 0.785$ s, $w_C = 2.94\ \mu J$

and

At $t = 1.57$ s, $w_C = 2.54\ \mu J$

15. KCL yields

$$\cos 6t + 0.2 i_L = i_L + \frac{v_L}{10}$$

where

$$v_L = L\frac{di_L}{dt}$$

so

$$\cos 6t = 0.8 i_L + 0.5\frac{di_L}{dt} \quad [1]$$

$$i_L(t) = I_1 \cos 6t + I_2 \sin 6t$$

$$\frac{di_L}{dt} = -6I_1 \sin 6t + 6I_2 \cos 6t \quad [2]$$

Substituting Eq [2] into Eq [1],

$$\cos 6t = 0.8I_1 \cos 6t + 0.8I_2 \sin 6t + 3I_2 \cos 6t - 3I_1 \sin 6t$$

We can use this to generate two equations in two unknowns:

$$0.8I_1 + 3I_2 = 1$$

$$-3I_1 + 0.8I_2 = 0$$

Solving,

$$I_1 = 0.083 \text{ A}$$

$$I_2 = 0.31 \text{ A}$$

Thus,

$$i_L(t) = 0.321\cos(6t - 75°) \text{ A}$$

Then,

$$v_L = L\frac{di_L}{dt} = -(0.5)(0.321)(6)\sin(6t - 75°)$$

$$= -0.963\sin(6t - 75°) = 0.963\sin(6t + 105°) \text{ V } = v_R$$

Hence,

$$P_R = \frac{1}{10}v_L^2 = 0.0927\sin^2(6t + 105°) \text{ W}$$

16.

(a) $50\cos(-75°)+j\sin(-75°)=\boxed{12.94-j48.30}$

(b) $19\angle 30°=\boxed{16.45+j9.5}$

(c) $2.5\angle -30°+0.5\angle 45°=\boxed{2.52-j0.896}$

(d) $(2+j2)(2-j2)=8+j0=\boxed{8\angle 0}$

(e) $(2+j2)(5\angle 22°)=(2.83\angle 45°)(5\angle 22°)=\boxed{14.15\angle 67°}$

17. (a) $1+e^{j45°} = 1+1\angle 45° = 1+0.707+j0.707 = \boxed{1.85\angle 22.5°}$

(b) $-j\left(j\right)^2 = +j = \boxed{1\angle 90°}$

(c) $32 = \boxed{32\angle 0°}$

(d) $2-e^{j45°} = 2-1\angle 45° = 2-0.707-j0.07 = \boxed{1.293-j0.707}$

(f) $-j+5\angle 0 = -j+5 = \boxed{5-j}$

18. (a) $4(8-j8)=(4\angle 0)(11.31\angle -45°)=\boxed{43.25\angle -45°}$

(b) $4\angle 5°-2\angle 15°=3.98+j0.349-1.93-j0.518=\boxed{2.06\angle -4.71°}$

(c) $2+j9-5\angle 0=2+j9-5=-3+j9=\boxed{9.49\angle 108°}$

(d)

$$\frac{-j}{10+j5}-3\angle 40°+2=\frac{-j(10-j5)}{(10+j5)(10-j5)}-3\angle 40°+2$$
$$=0.04-j0.08-2.30-j1.93+2=\boxed{2.04\angle -99.6°}$$

19. (a) $3(3\angle 30°) = 9\angle 30° = \boxed{7.79 + j4.5}$

(b) $2\angle 25° + 5\angle -10° = 1.81 + j0.845 + 4.92 - j0.868 = \boxed{6.73 - j0.023}$

(c) $12 + j90 - 5\angle 30° = 12 + j90 - 4.33 + j2.5 = \boxed{16.33 + j92.5}$

(d) $\dfrac{10 + j5}{8 - j} + 2\angle 60° + 1 = 1.15 + j0.769 + 1 + j1.73 + 1 = \boxed{3.15 + j2.5}$

20. (a)

$$\frac{2+j3}{1.8\angle 90°}-4=\frac{3.61\angle 56.3}{1.8\angle 90°}-4=2.01\angle 26.3°-4$$
$$=1.798+j0.889-4=\boxed{2.37\angle 158°}$$

$$\frac{10\angle 25°}{5\angle -10°}+\frac{3\angle 15°}{3-j5}=2\angle 35°+0.514\angle 74°$$

(b) $=1.64+j1.15+0.142+j0.494=1.782+j1.644$

$$\boxed{=2.42\angle 42.7°}$$

(c) $$\frac{(1-j)(1+j)+1}{-j}\left(3\angle -90°\right)+\frac{j}{1\angle -45°}=\left(\frac{3\angle 0}{1\angle -90°}\right)\left(3\angle -90°\right)+\frac{1\angle 90°}{1\angle -45°}$$
$$=9\angle 0+1\angle 138°=8.32\angle 4.87°=\boxed{8.29+j0.707}$$

21.

$5\sin 20t \text{ V} \rightarrow 5e^{j20t} \text{ V}$

$KVL:\ -5e^{j20t} + 5i_C + v_C = 0$

$where\ i_C = C\dfrac{dv_C}{dt} = 0.13\dfrac{dv_C}{dt}.$

$Thus,$

$$5(0.13)\frac{dv_C}{dt} + v_C = 5e^{j20t} \qquad [1]$$

Substituting

$v_C = V_m e^{j(20t+\phi)}$

into Eq. [1],

$5(0.13)V_m(j20)e^{j(20t+\phi)} + V_m e^{j(20t+\phi)} = 5e^{j20t}$

$(j13V_m + V_m)e^{j(20t+\phi)} = 5e^{j20t}$

Solving for V_m,

$V_m\sqrt{1+13^2} = 5$ or $V_m = 0.383$ V

and

$$\phi + \tan^{-1}\frac{13}{1} = 0 \therefore \phi = -85.6°$$

$$v_C(t) = 0.383\sin(20t - 85.6°) \text{ V}$$

Thus, and

$$i_C = C\frac{dv_C}{dt} = (0.13)(0.383)(20)\cos(20t - 85.6°) = 0.996\cos(20t - 85.6°) \text{ A}$$

22. $i_s = 2\cos 5t \text{ A} \rightarrow 2\text{e}^{j5t} \text{ A}$

$$v_L = 2(i_s - i_L) = L\frac{di_L}{dt}$$

so

$$4e^{j5t} - 2i_L - 0.4\frac{di_L}{dt} = 0$$

Using

$$i_L(t) = \text{I}_m\, e^{j(5t+\phi)}$$

$$\frac{di_L}{dt} = j5\text{I}_m\, e^{j(5t+\phi)}$$

and substituting into our circuit equation,

$$2I_m e^{j(5t+\phi)} + j2I_m e^{j(5t+\phi)} = 4e^{j5t}$$

Next, we note that

$$I_m\sqrt{2^2+2^2} = 4,\ \text{so } I_m = 1.41 \text{ A}$$

and

$$\phi + \tan^{-1}\left(\frac{2}{2}\right) = 0 \therefore \phi = -45°$$

Finally, we may write

$$\boxed{i_L(t) = 1.41\cos(5t - 45°) \text{ A}}$$

23. $i_L = 1.8e^{j(5t+26.6°)}$, so we may write:

$$2I_m e^{j(5t+\phi)} + j2I_m e^{j(5t+\phi)} = 1.8e^{j(5t+26.6°)}$$

Note that

$$I_m\sqrt{2^2+2^2} = 1.8 \therefore I_m = 0.636 \text{ A}$$

$$\phi + \tan^{-1}\left(\frac{2}{2}\right) = 26.6°, \text{ so } \phi = -18.4°$$

Consequently,

$$i_L(t) = 0.636\cos(5t - 18.4°) \text{ A}$$

$$v_L(t) = L\frac{di_L}{dt} = 0.4(0.636)(5)\cos(5t-18.4°)$$

$$= 1.272\cos(5t-18.4°) \text{ V}$$

and

$$i_s(t) = i_L(t) + \frac{v_L}{2} = 0.636\cos(5t-18.4°) + 0.636\cos(5t-18.4°)$$

$$= \boxed{1.272\cos(5t-18.4°) \text{ A}}$$

24.

$i_L = I_m e^{j(25t+\phi)}$

$5\sin(35t - 10°)\text{ A} \rightarrow 5e^{j(35t-10°)}\text{ A}$

$i_L = 5e^{j(35t-10°)} - i_C \qquad [1]$

where

$i_C = C\dfrac{dv_C}{dt} = 0.01\dfrac{dv_C}{dt}$

$$v_C = 6i_L + v_L = 6i_L + 0.4\frac{dv_C}{dt}$$

so

$$\frac{dv_C}{dt} = 6(j35)I_m e^{j(35t+\phi)} + 0.4(j35)e^{j(35t+\phi)}$$

and Eq. [1] *becomes*

Note that $I_m e^{j(35t+\phi)} = 5e^{j(35t-10°)} - j2.1I_m e^{j(35t+\phi)} + 4.9e^{j(35t+\phi)}$

Thus,

$$5 = I_m\left[(1-4.9)^2 + (2.1)^2\right]^{\frac{1}{2}} \quad \text{or} \quad I_m = 5/4.429 = 1.13\text{ A}$$

and

$$-10° = \phi + \tan^{-1}\left(\frac{2.1}{1-4.9}\right) \quad \text{or} \quad \phi = -38.3°$$

Finally,

$$\boxed{i_L(t) = 1.13\cos(35t + 18.3°)\text{ A}} = 1.13\sin(35t - 162°)\text{ A}$$

25. (a) $\boxed{28\angle 0}$

(b) $32\angle\left(-90° - 90°\right) = \boxed{32\angle -180°}$

(c) $1\angle\left(45° - 90°\right) = \boxed{1\angle -45°}$

(d)

$$5\cos 10t + 8\cos(10t + 45°) = 5\cos 10t + 8\left[\cos 45° \cos 10t - \sin 45° \sin 10t\right]$$

$$= 5\cos t + 4\sqrt{2}\cos 10t - 4\sqrt{2}\sin 10t = (5 + 4\sqrt{2})\cos 10t - 4\sqrt{2}\sin 10t$$

$$= \sqrt{\left(5 + 4\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2}\cos\left(10t + \tan^{-1}\frac{4\sqrt{2}}{5 + 4\sqrt{2}}\right)$$

$$= \boxed{12.1\cos(10t + 28°)}$$

26. (a) $11\angle -90°$

(b) $11\angle 0$

(c) $11\angle -90°$

(d) $3\cos 100t - 3\sin 100t = A\cos\left(100t + \tan^{-1}\frac{3}{3}\right) = 4.24\cos(100t + 45°)$

27. $\omega = 2\pi f = 2\pi(1000) = 6280$ rad/s

(a) $9\angle 65°\text{ V} \rightarrow \boxed{9\cos(6280t + 65°)\text{ V}}$

(b) $\dfrac{2\angle 31°}{4\angle 25°}\text{ A} = 0.5\angle 6° \rightarrow \boxed{0.5\cos(6280t + 6°)\text{ A}}$

(c)

$$22\angle 14° - 8\angle 33°\text{ V} = 21.35 + j5.32 - 6.71 - j4.36 = 14.64 - j0.96$$
$$= 14.67\angle -3.75°\text{ V}$$
$$\rightarrow \boxed{14.67\cos(6280t - 3.75°)\text{ V}}$$

28. (a) $\dfrac{2-j}{5\angle 45°}\ \text{V} = \dfrac{2.24\angle -26.6°}{5\angle 45°} = \boxed{0.448\angle -71.6°\ \text{V}}$

(b) $\dfrac{6\angle 20°}{1000} - j\ \text{V}\ = 0.006\angle 20° - j = 0.00564 + j0.00205 - j$

$= 0.00564 - j0.998 = \boxed{0.998\angle 89.7°\ \text{V}}$

(c) $(j)(52.5\angle -90°) = (1\angle 90°)(52.5\angle -90°) = \boxed{52.5\angle 0\ \text{V}}$

29. f = 50 Hz so $\omega = 2\pi f = 314$ rad/s

(a) $\dfrac{2-j}{5\angle 45°} = \dfrac{2.24\angle -26.6°}{5\angle 45°} = 0.448\angle -71.6°$ V

Thus,

$v(t) = 0.448\cos(314t - 71.6°)$ V

$v(0.01) = -0.141$ V

$v(0.025) = 0.426$ V

(b) $\dfrac{2\angle 31°}{4\angle 25°} = 0.998\angle -89.7°$ V

Thus,

$v(t) = 0.998\cos(314t - 89.7°)$ V

$v(0.01) = -0.00364$ V

$v(0.025) = 0.998$ V

(c)

$(j)(52.5\angle -90°) = 52.5\angle 0$

so

$v(t) = 52.5\cos(314t + 0) = 52.5\cos 314t$ V

and

$v(0.01) = -52.5$ V

$v(0.025) = 0.209$ V

30. f = 50 Hz so $\omega = 2\pi f = 314$ rad/s

(a)

$$v(t) = 9\cos(314t + 65°) \text{ V}$$

$\therefore$

$$v(0.01) = -3.82 \text{ V}$$
$$v(0.025) = -8.14 \text{ V}$$

(b)

$$i(t) = 0.5\cos(314t + 6°) \text{ A}$$

$\therefore$

$$i(0.01) = -0.497 \text{ A}$$
$$i(0.025) = -0.050 \text{ A}$$

(c)

$$v(t) = 14.67\cos(314t - 3.75°) \text{ V}$$

$$v(0.01) = -14.6 \text{ V}$$
$$v(0.025) = 1.02 \text{ V}$$

31. $\omega = 5$ rad/s; $\mathbf{I} = 2\angle 0$ mA

(a) $\mathbf{V} = (1000)(0.002\angle 0) = \boxed{2\angle 0 \text{ V}}$

(b) C = 0.001 F

$$\mathbf{V} = \frac{\mathbf{I}}{j\omega C} = \frac{0.002\angle 0}{j(5)(0.001)} = \frac{0.002\angle 0}{0.005\angle 90^\circ} = \boxed{0.4\angle -90^\circ \text{ V}}$$

(c) L = 10^{-9} H

$$\mathbf{V} = j\omega L\mathbf{I} = j(5)(10^{-9})(0.002\angle 0) = \boxed{10^{-4}\angle 90^\circ \text{ V}}$$

32. $\omega = 1$ rad/s

(a) *If* $\mathbf{V}_R = 1\angle 30°$ V, R = 1 Ω, then $\mathbf{I}_R = \mathbf{I}_C = \mathbf{I}_L = \boxed{1\angle 30° \text{ A}}$

(b)

$$\mathbf{V}_L = j\omega L\mathbf{I} = j(1)(1)\mathbf{I} = (1\angle 90°)(1\angle 30°) = 1\angle 120° \text{ V}$$

$$\mathbf{V}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1\angle 30°}{j(1)(1)} = \left(1\angle 30°\right)\left(1\angle -90°\right) = 1\angle -60° \text{ V}$$

$$\text{Ratio} = \frac{1\angle 30°}{1\angle 120° + 1\angle -60°} = \frac{1\angle 30°}{1.5\angle 119°} = \boxed{0.667\angle -89°}$$

(c)

$\omega = 2$ rad/s $\therefore$

$\mathbf{V}_L = 2\angle 120°$ V

$\mathbf{V}_C = 0.5\angle -60°$ V

$\therefore$

$$\text{Ratio} = \frac{1\angle 30°}{2\angle 130° + 0.5\angle -60°} = \frac{1\angle 30°}{0.51\angle 133°} = \boxed{0.662\angle -103°}$$

33. $\omega = 314$ rad/s. $\mathbf{I} = 10\angle 0$ mA $= 0.01\angle 0$ A

(a) 2 Ω: $\mathbf{V}_R = 2\mathbf{I} = \boxed{0.02\angle 0 \text{ V}}$

(b) 1 F: $\mathbf{V}_C = \dfrac{\mathbf{I}}{j\omega C} = \dfrac{0.01\angle 0}{j(314)(1)} = \boxed{3.18\times 10^{-5}\angle -90° \text{ V}}$

(c) 1 H: $\mathbf{V}_L = j\omega L\mathbf{I} = j(314)(1)(0.01\angle 0) = \boxed{3.14\angle 90° \text{ V}}$

(d) 2 Ω, 1 F:

$$\mathbf{V}_R = 0.02\angle 0 \text{ V}$$
$$\mathbf{V}_C = 3.18\times 10^{-5}\angle -90° \text{ V}$$
$$\mathbf{V}_{TRC} = \mathbf{V}_R + \mathbf{V}_C = \boxed{0.02\angle -90° \text{ V}}$$

(e) 2 Ω, 1 H:

$$\mathbf{V}_R = 0.02\angle 0 \text{ V}$$
$$\mathbf{V}_L = 3.14\angle 90° \text{ V}$$
$$\mathbf{V}_{TRL} = \mathbf{V}_R + \mathbf{V}_C = \boxed{3.14\angle 90° \text{ V}}$$

(f)

$$v_R(0) = 0.02\cos(0) = 0.02 \text{ V}$$
$$v_C(0) = 3.18\times 10^{-5}\cos(-90°) = 0 \text{ V}$$
$$v_L(0) = 3.14\cos(90°) = 0 \text{ V}$$
$$v_{TRC}(0) = 0.02\cos(-0.09°) = 0.02 \text{ V}$$
$$v_{TRL}(0) = 3.14\cos(89.6°) = 0.0219 \text{ V}$$

34. $\mathbf{I}_{10} = 0.002\angle 42°$ A
$\mathbf{I} = 0.00141\angle -3°$ A

(a)

$$\mathbf{I}_{10} - \mathbf{I} = \mathbf{I}_{25\Omega} = 0.00141\angle 86.6° \text{ A}$$
$$\mathbf{V}_{25\Omega} = 25\mathbf{I}_{25\Omega} = 0.0354\angle -3° \text{ V}$$
$$\frac{\mathbf{V}_{25\Omega}}{\mathbf{I}_{25\Omega}} = \frac{0.0354\angle 86.6°}{0.0354\angle -3°} = 25.1\angle 89.8° \ \Omega$$

Since the voltage leads the current by essentially 90°, it is likely an inductor.

$j\omega L = 25.1\angle 90°$

$\therefore$

(b) $\omega L = 25.1$

so

$$L = \frac{25.1}{\omega} = \frac{25.1}{1000} = 25.1 \text{ mH}$$

35. $\mathbf{I}_{10} = 4\angle 35°$ A ; $\mathbf{V} = 10\angle 35°$ V; $\mathbf{I} = 2\angle 35°$ A

(a) No phase shift exists between $\mathbf{I}_{10}$ and $\mathbf{V}$, so the element is likely a simple resistor:

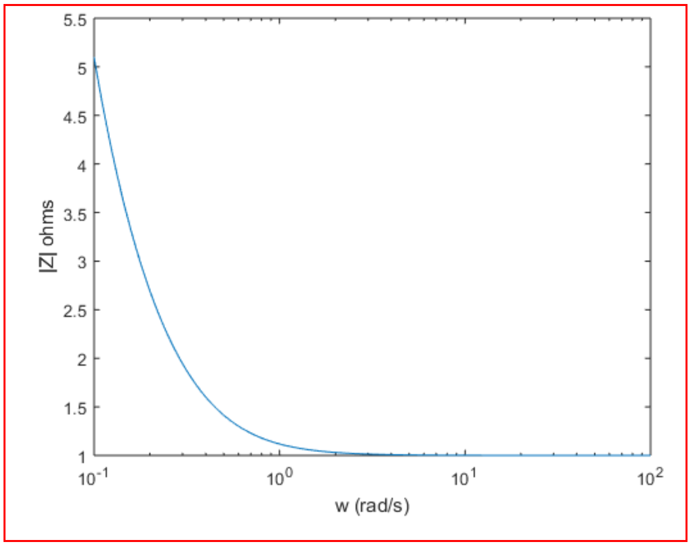
$$R = \frac{\mathbf{V}}{\mathbf{I}_{10}} = \frac{10\angle 35°}{4\angle 35°} = \frac{10}{4} = \boxed{2.5\ \Omega}$$

(b)

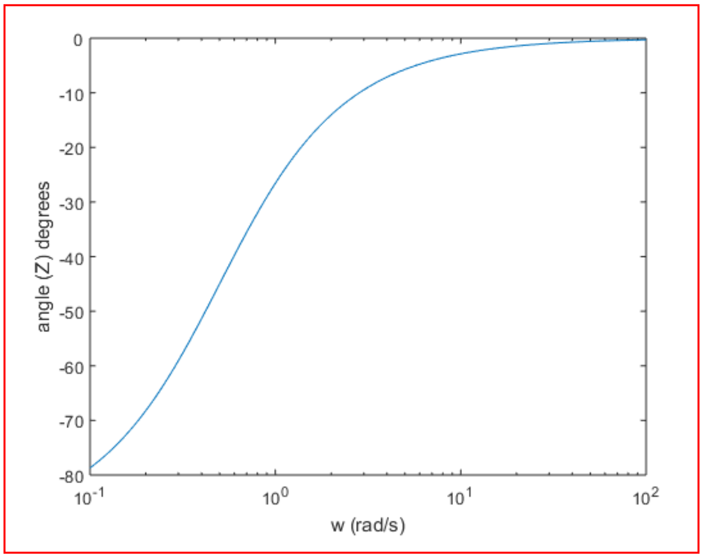
$$\mathbf{I}_{25} = \mathbf{I}_{10} - \mathbf{I} = 4\angle 35° - 2\angle 35° = 2\angle 35°\text{ A}$$
$$\mathbf{V}_{25} = 25(2\angle 35°) = 50\angle 35°\text{ V}$$
$$\mathbf{V}_s = 10\mathbf{I}_{10} + \mathbf{V} + \mathbf{V}_{25} = 40\angle 35° + 10\angle 35° + 50\angle 35°$$
$$= \boxed{100\angle 35°\text{ V}}$$

36. (a) $\mathbf{Z}_{eq} = \boxed{1 + \dfrac{1}{j2\omega}\ \Omega}$

(b)
```
>> w=logspace(-1,2,500);
Zeq = 1 + 0.5/i./w;
Zmag=abs(Zeq);
Zang=angle(Zeq)*180/pi;
>> semilogx(w,Zang);xlabel('w (rad/s)');ylabel('angle (Z) degrees');
```



(c)



37. $\omega = 20$ rad/s

(a) We have 1 kΩ || 1 mH:

$\mathbf{Z}_R = 1000\ \Omega$

$\mathbf{Z}_L = j\omega L = j(20)(10^{-3}) = j20\times10^{-3}\ \Omega$

$$\mathbf{Z}_{eff} = \frac{(1000)(j20\times10^{-3})}{1000 + j20\times10^{-3}} = \boxed{0.02\angle 90^\circ\ \Omega}$$

(b)

$$\mathbf{Z}_s = j\omega L + \frac{1}{j\omega C} = j(20)(1) + \frac{1}{j(20)(1)} = 19.5\angle 90^\circ\ \Omega$$

$$\mathbf{Z}_{eq} = \frac{(10)(19.5\angle 90^\circ)}{10 + 19.5\angle 90^\circ} = \boxed{8.9\angle 27^\circ\ \Omega}$$

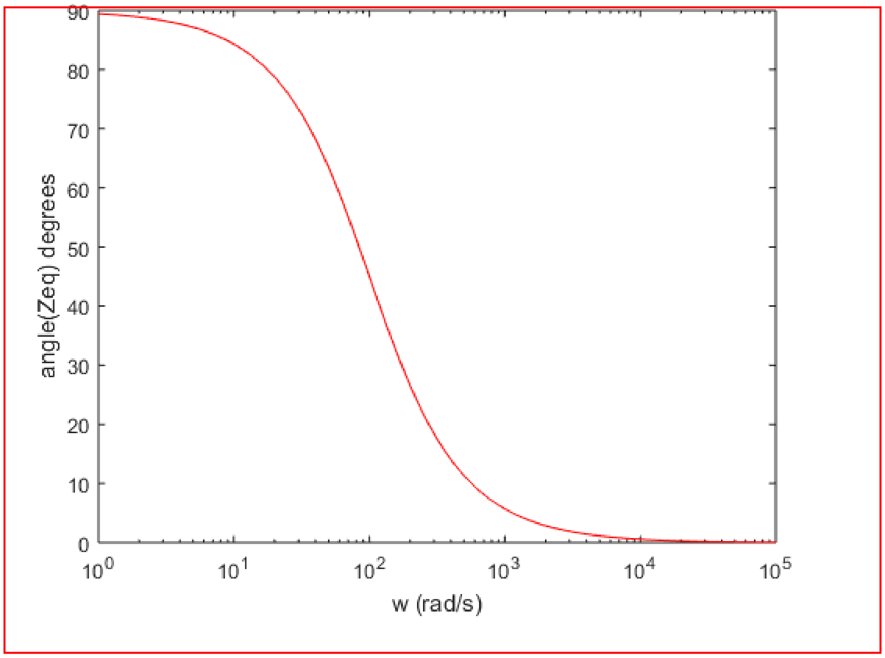
38. (a)

$$\mathbf{Z}_L = j\omega L = j\omega(0.01)$$

$$\mathbf{Z}_{eq} = \frac{(1)(j\omega 0.01)}{1 + j0.01\omega} = \boxed{\frac{j0.01\omega}{1 + j0.01\omega}\ \Omega}$$

(b)

```
>> w=logspace(0,5,500);
>> Zeq = i*0.01*w./(1+i*0.01*w);
>> Zang=angle(Zeq)*180/pi;
>> semilogx(w,Zang,'r-');xlabel('w (rad/s)');ylabel('angle(Zeq) degrees');
```



As a side note, as the angle of the effective impedance approaches zero, the impedance increasingly appears to be a simple resistance.

39. $\omega = 1000$ rad/s

(a)

$$\mathbf{Z}_L = j\omega L = j(1000)(0.02) = j0.02\ \Omega$$

$$\mathbf{Z}_{eq} = 25 + \mathbf{Z}_L = 25 + j20\ \Omega = 32\angle 38.7°\ \Omega$$

$$\mathbf{Y}_{eq} = \frac{1}{\mathbf{Z}_{eq}} = \boxed{0.031\angle -39°\ \text{S}}$$

(b) $$\mathbf{Y}_{eq} = \frac{1}{j20} + \frac{1}{25} = \boxed{0.064\angle -51.3°\ \text{S}}$$

(c) $$\mathbf{Y}_{eq} = \frac{1}{j20} + \frac{1}{25} + j\omega C = 0.064\angle -51.3°\ \text{S} + \text{j}(1000)(20\times 10^{-3})$$

$$\boxed{= 20\angle 90°\ \text{S}}$$

40.

$$\mathbf{Z}_1 = 55 \,||\, j\omega L = \frac{(55)(j0.02\omega)}{55 + j0.02\omega}$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} + 20 = 20 - \frac{j100}{\omega} = \frac{20\omega - j100}{\omega}$$

$$\mathbf{Z}_{eq} = 25 \,||\, (\mathbf{Z}_1 + \mathbf{Z}_2)$$

(a)

$\omega = 1$ rad/s

$\mathbf{Z}_1 = 0.02\angle 90° \ \Omega$

$\mathbf{Z}_2 = 102\angle -79° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{(25)(0.02\angle 90° + 102\angle -79°)}{25 + 0.02\angle 90° + 102\angle -79°} = \boxed{23.3\angle -13° \ \Omega}$$

(b)

$\omega = 10$ rad/s

$\mathbf{Z}_1 = 0.2\angle 88° \ \Omega$

$\mathbf{Z}_2 = 22.4\angle -26.6° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{(25)(0.2\angle 88° + 22.4\angle -26.6°)}{25 + 0.2\angle 88° + 22.4\angle -26.6°} = \boxed{12.1\angle -13.8° \ \Omega}$$

(c)

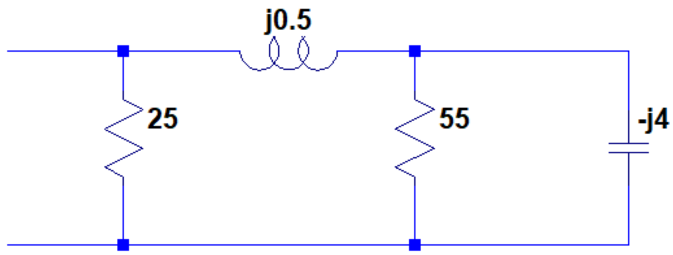
$\omega = 100$ rad/s

$\mathbf{Z}_1 = 2\angle 88° \ \Omega$

$\mathbf{Z}_2 = 20.\angle -3° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{(25)(2\angle 88° + 20.\angle -3°)}{25 + 2\angle 88° + 20.\angle -3°} = \boxed{11.1\angle 1.5° \ \Omega}$$

41. Redrawing as directed, we obtain (all quantities shown in ohms):



$$j\omega L = j(25)(0.02) = j0.5\ \Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j(250(0.01)} = -j4\ \Omega$$

$$\mathbf{Z}_1 = 55\,||\,(-j4) = 4\angle -86°\ \Omega$$

$$\mathbf{Z}_2 = 20 + j0.5 = 20\angle 1.4°\ \Omega$$

$$\mathbf{Z}_{eq} = 25\,||\,(\mathbf{Z}_1 + \mathbf{Z}_2) = 25\,||\,\angle -10°\ \Omega$$

Thus,

$$\mathbf{Z}_{eq} = \boxed{25\,||\,\angle -5.5°\ \Omega}$$

42. $\mathbf{I} = 3\angle -20°$ A, $\omega = 2000$ rad/s

(a)

$$\mathbf{Z} = 3 + j\omega L = 3 + j(2000)(0.002) = 3 + j4\ \Omega$$

$$\mathbf{V} = \mathbf{Z} \cdot \mathbf{I} = (3\angle -20°)(3 + j4) = (3\angle -20°)(5\angle 53°) = 15\angle 33°\ \Omega$$

(b)

$$\mathbf{Z} = 3 - \frac{j}{\omega C} = 3 - \frac{j}{(2000)(125\times 10^{-6})} = 5\angle -53°\ \Omega$$

$$\mathbf{V} = \mathbf{Z} \cdot \mathbf{I} = \left(5\angle -53°\right)\left(3\angle -20°\right) = \boxed{15\angle -73°\ \text{V}}$$

(c)

$$\mathbf{Z} = 3 + j\omega L + \frac{1}{j\omega C} = 3 + 3 + j4 + 4\angle 90° = 10\angle 53°\ \Omega$$

$$\mathbf{V} = \mathbf{Z} \cdot \mathbf{I} = \left(3\angle -20°\right)\left(10\angle 53°\right) = \boxed{30\angle 33°\ \text{V}}$$

(d) $\omega = 4000$ rad/s

$$\mathbf{Z}_R = 3\ \Omega$$

$$\mathbf{Z}_L = 3 + j(4000)(0.002) = 3 + j8\ \Omega$$

$$\mathbf{Z}_C = \frac{-j}{(4000)(125\times 10^{-6})} = 2\angle -90°\ \Omega$$

$$\mathbf{Z} = \mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C = 3 + 3 + j8 - j2 = 6 + j6 = 8.49\angle 45°\ \Omega$$

$$\mathbf{V} = \mathbf{Z} \cdot \mathbf{I} = \left(3\angle -20°\right)\left(8.49\angle 45°\right) = \boxed{25.5\angle 25°\ \text{V}}$$

43.

$$\mathbf{Z}_1 = 60 \,||\, \frac{-j}{\omega C} = 60 \,||\left[\frac{-j33.3\times10^{-3}}{\omega}\right] = (60)\left(\frac{\frac{-j33.3\times10^{-3}}{\omega}}{60-\frac{j33.3\times10^{-3}}{\omega}}\right)$$

$$\mathbf{Z}_2 = 60 \,||\, j\omega L = 60 \,||\left[j\omega 0.01\right] = (60)\left(\frac{j\omega 0.01}{60+j\omega 0.01}\right)$$

$$\mathbf{Z}_{eq} = 60 \,||\left(\mathbf{Z}_1 + \mathbf{Z}_2\right)$$

(a)

$f = 1 \text{ Hz} \rightarrow \omega = 2\pi f = 3.14 \text{ rad/s}$

$\mathbf{Z}_1 = 60\angle -0.3° \ \Omega$

$\mathbf{Z}_1 = 0.031\angle 90° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{60(60\angle -0.3° + 0.031\angle 90°)}{60 + 60\angle -0.3° + 0.031\angle 90°} \ \Omega = \boxed{30\angle -0.15° \ \Omega}$$

(b)

$\omega = 2\pi f = 6280 \text{ rad/s}$

$\mathbf{Z}_1 = 5.28\angle -85° \ \Omega$

$\mathbf{Z}_1 = 43.4\angle 43.7° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{60(5.28\angle -85° + 43.4\angle 43.7°)}{60 + 5.28\angle -85° + 43.4\angle 43.7°} \ \Omega = \boxed{25.4\angle 22.8° \ \Omega}$$

(c)

$\omega = 2\pi f = 6.28\times10^6 \text{ rad/s}$

$\mathbf{Z}_1 = 0.005\angle -90° \ \Omega$

$\mathbf{Z}_1 = 60\angle 0.05° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{60(0.005\angle -90° + 60\angle 0.05°)}{60 + 0.005\angle -90° + 60\angle 0.05°} \ \Omega = \boxed{30\angle 0.02° \ \Omega}$$

(d)

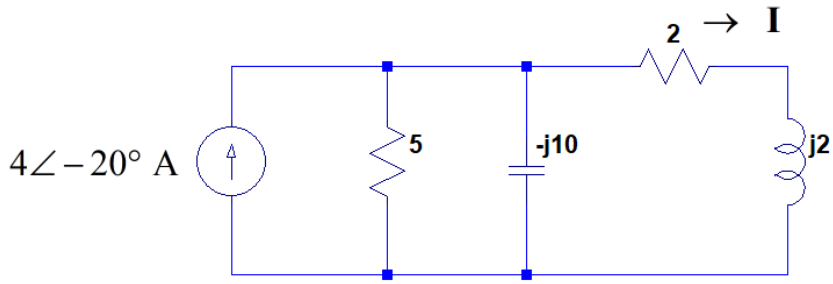
$\omega = 2\pi f = 6.28\times10^9 \text{ rad/s}$

$\mathbf{Z}_1 = 5.3\times10^{-6}\angle -90° \ \Omega$

$\mathbf{Z}_1 = 60\angle 0° \ \Omega$

$$\mathbf{Z}_{eq} = \frac{60(5.3\times10^{-6}\angle -90° + 60\angle 0°)}{60 + 5.3\times10^{-6}\angle -90° + 60\angle 0°} \ \Omega = \boxed{30\angle 0° \ \Omega}$$

44. Drawing the circuit,



The current **I** flows through the series combination 2 + *j*2 Ω. Thus, by current division,

$$\mathbf{I} = (4\angle -20^\circ)\frac{\frac{1}{2+j2}}{5-j10+\frac{1}{2+j2}} = (4\angle -20^\circ)(0.031\angle 17.9^\circ)$$

$= 0.123\angle -2^\circ$ A

Thus,

$i(t) = 0.123\cos(100t - 2^\circ)$ A

45. **One possible set of solutions from many (design problem, so no unique solution)**:
Series combination of resistor (R), inductor (L), and capacitor (C). $\omega = 100$ rad/s (given).

(a) Set R = 1 Ω.

$$j\omega L - \frac{j}{\omega C} = 0$$

or

Then, we need $\omega^2 = \frac{1}{LC} = (100)^2$

Setting L = 1 mH,

C = 0.1 F

(b)

$$R + j\omega L - \frac{j}{\omega C} = 7\angle 10° = 6.89 + j1.22\ \Omega$$

$$\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} = 7$$

or

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 49$$

Solving,

$$\left(\omega L - \frac{1}{\omega C}\right) = 1.22$$

Set L = 100 mH. Then C = 1.14 mF and R = 6.89 Ω

(c)

$$R + j\omega L - \frac{j}{\omega C} = 5\angle -53.1° = 3 - j4 =\ \Omega$$

$$\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} = 5$$

Solving,

$$\left(\omega L - \frac{1}{\omega C}\right) = -4$$

Set R = 3 Ω and L = 1 mH. Then C = 2.44 mF.

46. **One possible set of solutions from many (design problem, so no unique solution)**:
Parallel combination of resistor (R), inductor (L), and capacitor (C). $\omega = 10$ rad/s (given).

(a) We need

$$\mathbf{Y} = 1\text{ S} = \frac{1}{R} + j\omega C - \frac{j}{\omega L}$$

$Setting\ R = 1\ \Omega,$

$$j\omega C - \frac{j}{\omega L} = 0$$

$$or\ \omega^2 = \frac{1}{LC}.$$

Set L = 1 mH. Then, C = 10 F.

This is not very realistic! Try C = 100 mF. Then, L = 100 mH.

(b)

$$\mathbf{Y} = 12\angle -18°\text{ S} = 11.41 - j3.71\text{ S} = \frac{1}{R} + j\omega C - \frac{j}{\omega L}$$

$$Setting\ R = \frac{1}{11.41} = 0.0876\ \Omega,$$

$$j\omega C - \frac{j}{\omega L} = j\left(\omega C - \frac{1}{\omega L}\right) = -j3.71$$

Set C = 1 mF. Then, L = 26.2 mH.

(c)

$$\mathbf{Y} = 2 + j\text{ mS}$$

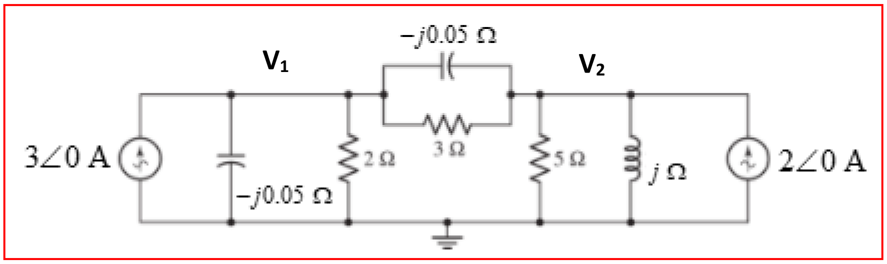
$$Set\ \frac{1}{R} = 0.002\text{ S} \therefore R = 500\ \Omega$$

Then

$$0.001j = j\omega C - \frac{1}{j\omega L}.$$

Set C = 100 mF. Then, L = 100 mH

47. (a) Redrawing the circuit in the phasor domain,



(b) $$3\angle 0 = \frac{\mathbf{V}_1}{-j25} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j0.05} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{3} \qquad [1]$$

$$2\angle 0 = \frac{\mathbf{V}_2}{j} + \frac{\mathbf{V}_2}{5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{3} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j0.05} \qquad [2]$$

Rearranging,

$$\left(j0.04 + 0.5 + j20 + 0.33\right)\mathbf{V}_1 - (0.33 + j20)\mathbf{V}_2 = 3 \qquad [1]$$

$$-(0.33 + j20)\mathbf{V}_1 + (-j + 0.2 + 0.33 + j20)\mathbf{V}_2 = 2 \qquad [2]$$

Solving,

$\mathbf{V}_1 = 4.11\angle 54.7°$ V

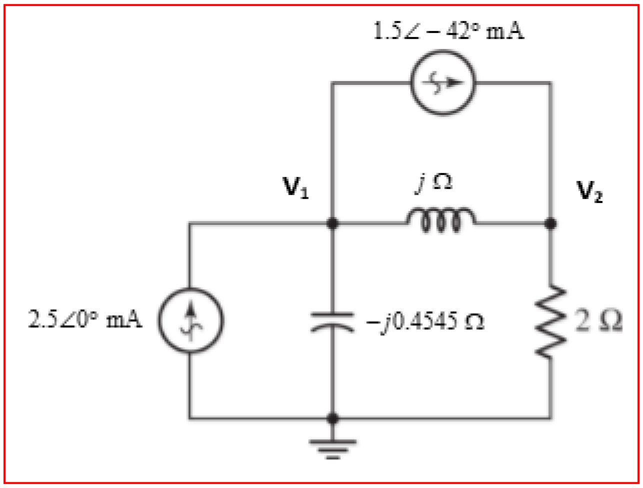
$\mathbf{V}_2 = 4.24\angle 54.5°$ V

Thus,

$v_1(t) = 4.11\cos(10t + 54.7°)$ V

$v_2(t) = 4.24\cos(10t + 54.5°)$ V

48. (a) Redrawing in the phasor domain,



(b) We need only define one clockwise mesh current $\mathbf{I}_3$ in the lower right mesh. Then,

$$(-j0.4545)(\mathbf{I}_3 - 0.0025\angle 0) + j(\mathbf{I}_3 - 0.0015\angle -42°) + 2\mathbf{I}_3 = 0$$

Solving,

$$\mathbf{I}_3 = \frac{-(+j0.4545)(0.0025) + (j)(0.0015\angle -42°)}{-j0.454 + j + 2} = 4.28\times 10^{-4}\angle -26°\ \Omega$$

Thus,

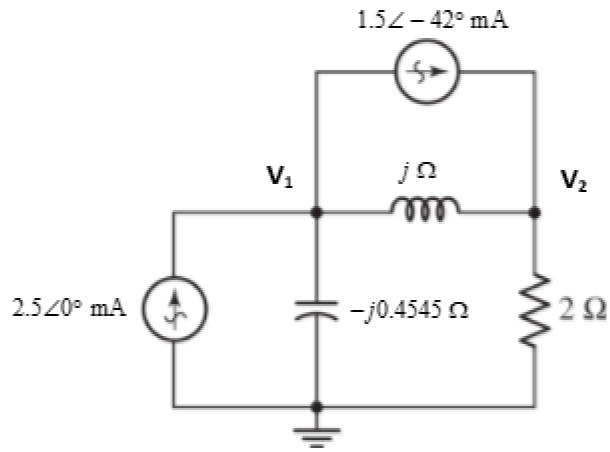
$i_1(t) = 2.5\cos 10t$ mA by inspection

$i_2(t) = 1.5\cos(10t - 42°)$ mA by inspection

and

$i_3(t) = 0.43\cos(10t - 26°)$ mA

49. We begin by redrawing in the phasor domain:



We require two nodal equations:

$$0.0025\angle 0 - 0.0015\angle -42° = \frac{\mathbf{V}_1}{-j0.4545} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j} \qquad [1]$$

$$0.0015\angle -42° = \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j} \qquad [2]$$

Or

$$(j2.2 - j)\mathbf{V}_1 + \qquad j\mathbf{V}_2 = 0.00171\angle 35.9° \qquad [1]$$

$$j\mathbf{V}_1 + (0.5 - j)\mathbf{V}_2 = 0.0015\angle -42° \qquad [2]$$

Solving,

$\mathbf{V}_1 = 0.878\angle 32.7°\ \text{V}$

$\mathbf{V}_2 = 1.05\angle -147°\ \text{V}$

So,

$v_1(t) = 0.878\cos(10t + 32.7°)\ \text{V}$

$v_2(t) = 1.05\cos(10t - 147°)\ \text{V}$

50. By mesh analysis,

$$-10\angle -80° + j30\mathbf{I}_1 + 55(\mathbf{I}_1 - \mathbf{I}_2) + 4\angle 0 = 0 \quad [1]$$

$$-4\angle 0 + 55(\mathbf{I}_2 - \mathbf{I}_1) - j20\mathbf{I}_2 + 2\angle -23° = 0 \quad [2]$$

Grouping terms,

$$(55 + j30)\mathbf{I}_1 - 55\mathbf{I}_2 = 4 + 10\angle -80° \quad [1]$$

$$-55\mathbf{I}_1 + (55 - j20)\mathbf{I}_2 = 4 - 2\angle -23° \quad [2]$$

Solving,

$$\mathbf{I}_1 = 0.808\angle -111° \text{ A}$$

$$\mathbf{I}_2 = 0.734\angle -89.1° \text{ A}$$

51. By mesh analysis,

$$-3\angle 0 + j30\mathbf{I}_1 + 55(\mathbf{I}_1 - \mathbf{I}_2) + 5.5\angle -130° = 0 \qquad [1]$$

$$-5.5\angle -130° + 55(\mathbf{I}_2 - \mathbf{I}_1) - j20\mathbf{I}_2 + 1.5\angle 17° = 0 \qquad [2]$$

Grouping terms,

$$(55 + j30)\mathbf{I}_1 - 55\mathbf{I}_2 = 7.78\angle 32.8° \qquad [1]$$

$$-55\mathbf{I}_1 + (55 - j20)\mathbf{I}_2 = 6.24\angle 142.8° \qquad [2]$$

Solving,

$$\mathbf{I}_1 = 0.433\angle 18.5° \quad \text{A}$$

$$\mathbf{I}_2 = 0.358\angle 52.8° \quad \text{A}$$

so

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{0.433\angle 18.5°}{0.358\angle 52.8°} = \boxed{1.21\angle -34.3°}$$

52. Begin by converting to phasor domain parameters:

$2.5\angle 9°\ \text{V}$

$j10\ \Omega$

$-j0.303\ \Omega$

Then, writing mesh equations:

$$-2.5\angle 9° + 2\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 0 \qquad [1]$$
$$j10(\mathbf{I}_2 - \mathbf{I}_1) - j0.303\mathbf{I}_2 + 5\mathbf{I}_1 = 0 \qquad [2]$$

Grouping terms,

$$(2 + j10)\mathbf{I}_1 - j10\mathbf{I}_2 = 2.5\angle 9° \qquad [1]$$
$$(5 - j10)\mathbf{I}_1 + j9.697\mathbf{I}_2 = 0 \qquad [2]$$

Solving,

$$\mathbf{I}_1 = 0.349\angle 11.5°\ \text{A}$$
$$\mathbf{I}_2 = 0.402\angle 38.1°\ \text{A}$$

Thus,

$$i_1(t) = 0.349\cos(10t + 11.5°)\ \text{A}$$
$$i_2(t) = 0.402\cos(10t + 38.1°)\ \text{A}$$

53. By nodal analysis, we may write

$$-5\angle -18° = \frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j3.8} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j4} \qquad [1]$$

$$2\angle 5° = \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j4} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1 + j3.8} \qquad [2]$$

Grouping terms,

$$\left[\frac{1}{j2} + \frac{1}{1 + j3.8} + \frac{j}{4}\right]\mathbf{V}_1 + \left[\frac{1}{j4} - \frac{1}{1 + j3.8}\right]\mathbf{V}_2 = -5\angle -18° \qquad [1]$$

$$\left[\frac{1}{j4} - \frac{1}{1 + j3.8}\right]\mathbf{V}_1 + \left[\frac{1}{2} + \frac{1}{1 + j3.8} + \frac{j}{4}\right]\mathbf{V}_2 = 2\angle 5° \qquad [2]$$

Solving,

$\mathbf{V}_1 = 9.60\angle -115.9°$ V

$\mathbf{V}_2 = 3.13\angle -10.1°$ V

And

$$\mathbf{I}_B = \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j3.8} = \boxed{2.77\angle 153° \text{ A}}$$

54. By nodal analysis, we may write

$$-15\angle 0° = \frac{\mathbf{V}_1}{j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1 + j3.8} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j4} \qquad [1]$$

$$25\angle 131° = \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j4} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{1 + j3.8} \qquad [2]$$

Grouping terms,

$$\left[\frac{1}{j2} + \frac{1}{1 + j3.8} + \frac{j}{4}\right]\mathbf{V}_1 - \left[\frac{1}{1 + j3.8} - \frac{1}{j4}\right]\mathbf{V}_2 = -15 \qquad [1]$$

$$\left[\frac{1}{j4} - \frac{1}{1 + j3.8}\right]\mathbf{V}_1 + \left[\frac{1}{2} + \frac{1}{1 + j3.8} + \frac{j}{4}\right]\mathbf{V}_2 = 25\angle 131° \qquad [2]$$

Solving,

$$\mathbf{V}_1 = 34.2\angle -103.5° \text{ V}$$
$$\mathbf{V}_2 = 41.91\angle 134.8° \text{ V}$$

55. Define $\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_x$ and apply mesh analysis:

$-4\angle 0 + (2+4.7)\mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) = 0$ [1]

$-(2+4.7-j5)\mathbf{I}_2 - 4.7\mathbf{I}_1 - 2\mathbf{I}_x = 0$ [2]

$(1+2+j2)\mathbf{I}_x - j2\mathbf{I}_1 + 2\mathbf{I}_2 = 0$ [3]

Solving,

$\mathbf{I}_x = 0.206\angle 81°$ A

so

$\mathbf{V}_x = (1)\mathbf{I}_x = 0.206\angle 81°$ V

and

$v_x(t) = 0.206\cos(20t + 81°)$ V

56. Define $\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_x$ and apply mesh analysis:

$-4\angle 0 + (2+4.7)\mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) = 0$ [1]

$-(2+4.7-j5)\mathbf{I}_2 - 4.7\mathbf{I}_1 - 2\mathbf{I}_x = 0$ [2]

$(1+2+j2)\mathbf{I}_x - j2\mathbf{I}_1 + 2\mathbf{I}_2 = 0$ [3]

Solving,

$\mathbf{I}_x = 0.206\angle 81°$ A

so

$i_x(t) = 0.206\cos(20t + 81°)$ A

57.

$\mathbf{V}_1 = 133\angle 77° \text{ V}$

$\mathbf{V}_2 = 55\angle 22° \text{ V}$

So

$-133\angle 77° + 0.8\mathbf{I}_1 + j0.312\mathbf{I}_1 - j0.392\mathbf{I}_2 = 0$ [1]

$(j0.392 - j0.714 + j0.448)\mathbf{I}_2 - j0.392\mathbf{I}_1 + j0.714\mathbf{I}_4 - j0.448\mathbf{I}_3 = 0$ [2]

$(0.6 + j0.448)\mathbf{I}_3 - j0.448\mathbf{I}_2 + 55\angle 22° = 0$ [3]

$(0.4 - j0.714)\mathbf{I}_4 + j0.714\mathbf{I}_2 = 0$ [4]

Solving,

$\mathbf{I}_1 = 165\angle 72.6° \text{ A}$

$\mathbf{I}_2 = 138\angle 73.5° \text{ A}$

$\mathbf{I}_3 = 148\angle 144.8° \text{ A}$

$\mathbf{I}_4 = 129\angle 115.6° \text{ A}$

and

$i_1(t) = 165\cos(14t + 72.6°) \text{ A}$

$i_2(t) = 138\cos(14t + 73.5°) \text{ A}$

$i_3(t) = 148\cos(14t + 144.8°) \text{ A}$

$i_4(t) = 129\cos(14t + 115.6°) \text{ A}$

58

$\mathbf{V}_1 = 0.009\angle 0.5°\ \text{V}$

$\mathbf{V}_2 = 0.004\angle 1.5°\ \text{V}$

$\omega = 500\ \text{rad/s}$

So

$28\ \text{mH} \rightarrow j14\ \Omega$

$100\ \text{mF} \rightarrow -j0.02\ \Omega$

$32\ \text{mH} \rightarrow j16\ \Omega$

Thus, defining $\mathbf{V}_3$ and $\mathbf{V}_4$,

$$0 = \frac{\mathbf{V}_3 - \mathbf{V}_1}{0.8} + \frac{\mathbf{V}_3}{j14} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{0.4} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{-j0.02} \qquad [1]$$

$$0 = \frac{\mathbf{V}_4 - \mathbf{V}_2}{0.6} + \frac{\mathbf{V}_4}{j16} + \frac{\mathbf{V}_4 - \mathbf{V}_3}{-j0.02} + \frac{\mathbf{V}_4 - \mathbf{V}_3}{0.4} \qquad [2]$$

Grouping terms,

$$-\frac{1}{0.8}\mathbf{V}_1 + \left(\frac{1}{0.8} + \frac{1}{j14} + \frac{1}{0.4} - \frac{1}{j0.02}\right)\mathbf{V}_3 + \left(\frac{1}{j0.02} - \frac{1}{0.4}\right)\mathbf{V}_4 = 0$$

$$-\frac{1}{0.6}\mathbf{V}_2 + \left(\frac{1}{j0.02} - \frac{1}{0.4}\right)\mathbf{V}_3 + \left(\frac{1}{0.6} + \frac{1}{j16} - \frac{1}{j0.02} + \frac{1}{0.4}\right)\mathbf{V}_4 = 0$$

Solving,

$\mathbf{V}_3 = 3.55\times 10^{-4}\angle -84.2°\ \text{V}$

$\mathbf{V}_4 = 1.30\times 10^{-4}\angle -83.8°\ \text{V}$

so

$v_3(t) = 3.55\times 10^{-4}\cos(500t - 84.2°)\ \text{V}$

$v_4(t) = 1.30\times 10^{-4}\cos(500t - 83.8°)\ \text{V}$

59. (a)

$$\mathbf{V}_o = A\mathbf{V}_i \ so \ \mathbf{V}_i = \frac{\mathbf{V}_o}{A} \quad [1]$$

Then,

$$\frac{\mathbf{V}_o - \mathbf{V}_i}{R_f}\left(\frac{-j}{\omega C_1}\right) = \left[\frac{\mathbf{V}_i - \mathbf{V}_s}{\frac{-j}{\omega C_1}}\right] \quad [2]$$

Using Eq. [1] to eliminate $\mathbf{V}_i$ in Eq. [2] yields

$$\mathbf{V}_o\left(\frac{j}{A\omega C_1} - \frac{j}{\omega C_1} - \frac{R_f}{A}\right) = \mathbf{V}_s R_f$$

Thus,

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{R_f}{\left[-\frac{R_f}{A} + j\left(\frac{1}{A\omega C_1} - \frac{1}{\omega C_1}\right)\right]}; \ As\ A \to \infty,\ \frac{\mathbf{V}_o}{\mathbf{V}_s} \to \frac{-R_f}{\frac{-j}{\omega C_1}} = -j\omega R_f C_1$$

(b)

$\mathbf{V}_o = A\mathbf{V}_i$ as before.

$$\mathbf{Z}_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega R_f C_f}$$

Then,

$$\left(\mathbf{V}_o - \mathbf{V}_i\right)\left(\frac{-j}{\omega C_1}\right) = \left(\mathbf{V}_i - \mathbf{V}_s\right)\mathbf{Z}_f$$

Eliminating $\mathbf{V}_i$ and collecting terms,

$$\mathbf{V}_o\left[\left(1 - \frac{1}{A}\right)\left(\frac{-j}{\omega C_1}\right) - \frac{\mathbf{Z}_f}{A}\right] = -\mathbf{V}_s\frac{\mathbf{Z}_f}{A}$$

Or, substituting in $\mathbf{Z}_f$,

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\frac{R_f}{1 + j\omega R_f C_f}}{\left(1 - \frac{1}{A}\right)\left(\frac{-j}{\omega C_1}\right) - \frac{1}{A}\left[\frac{R_f}{1 + j\omega R_f C_f}\right]}$$

$$As\ A \to \infty,\ \frac{\mathbf{V}_o}{\mathbf{V}_s} \to \frac{-\frac{R_f}{1 + j\omega R_f C_f}}{\frac{-j}{\omega C_1}} = -\frac{j\omega R_f C_1}{1 + j\omega R_f C_f}$$

60. 250 mF → $-j0.2\ \Omega$

70 mH → $j1.4\ \Omega$

$9\cos 20t \text{ V} \rightarrow 9\angle 0 \text{ V}$

$9\sin 20t \text{ V} \rightarrow 9\angle -90° \text{ V}$

Mesh equations:

$(3 - j0.2)\mathbf{I}_1 + j0.2\mathbf{I}_2 - 3\mathbf{I}_3 = 9\angle 0$

$0.005\mathbf{I}_1 + j0.2\mathbf{I}_1 + (j1.4 - j0.2)\mathbf{I}_2 - j1.4\mathbf{I}_4 = 0$

$-3\mathbf{I}_1 + (3 - j0.2)\mathbf{I}_3 + j0.2\mathbf{I}_4 = -9\angle -90°$

$-j1.4\mathbf{I}_2 + j0.2\mathbf{I}_3 + (5 + j1.4 - j0.2)\mathbf{I}_4 = 9\angle -90°$

Solving,

$\mathbf{I}_1 = 27.1\angle 132.5° \text{ A}$

$\mathbf{I}_2 = 6.14\angle -34° \text{ A}$

$\mathbf{I}_3 = 29.2\angle 132.5° \text{ A}$

$\mathbf{I}_4 = 1.82\angle -0.6° \text{ A}$

Hence,

$i_1(t) = 27.1\cos(20t + 132.5°) \text{ A}$

$i_2(t) = 6.14\cos(20t - 34°) \text{ A}$

$i_3(t) = 29.2\cos(20t + 132.5°) \text{ A}$

$i_4(t) = 1.82\cos(20t - 0.6°) \text{ A}$

61. Scaling to mA and kΩ, we can write two nodal equations:

$$3\angle -41^\circ = \frac{\mathbf{V}_{1a}}{3} - \frac{\mathbf{V}_{1a}}{j3} + \frac{\mathbf{V}_{1a} - \mathbf{V}_{2a}}{j8} + \frac{\mathbf{V}_{1a} - \mathbf{V}_{2a}}{3}$$

$$0 = \frac{\mathbf{V}_{2a}}{3} - \frac{\mathbf{V}_{2a}}{j2} + \frac{\mathbf{V}_{2a} - \mathbf{V}_{1a}}{-j5} + \frac{\mathbf{V}_{2a} - \mathbf{V}_{1a}}{j8}$$

Grouping terms and solving,

$$\mathbf{V}_{1a} = 4.20\angle -8.4^\circ \text{ V}$$
$$\mathbf{V}_{2a} = 0.584\angle 133.5^\circ \text{ V}$$

We now replace the $3\angle -41^\circ$ source with 0, and set the second equation to $-5\angle 13^\circ$
Then,

$$\mathbf{V}_{1b} = 0.973\angle -74.5^\circ \text{ V}$$
$$\mathbf{V}_{2b} = 9.12\angle -115^\circ \text{ V}$$

Note that
$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$$
$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b}$$

62. We can write two nodal equations:

$$33\angle 3^\circ - 51\angle -91^\circ = \frac{\mathbf{V}_1}{j3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5}$$

$$51\angle -91^\circ = \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5}$$

Which may be solved to yield

$$\mathbf{V}_1 = 391\angle 108.3^\circ \text{ mV}$$
$$\mathbf{V}_1 = 198\angle -156.7^\circ \text{ mV}$$

63. Here, we must employ superposition and recompute impedances each time.

$\mathbf{I}_1 = 0.004\angle 0$ A at $\omega = 40$ rad/s and $\mathbf{I}_1 = 0.004\angle -90°$ A at $\omega = 30$ rad/s

$j3 = j\omega L = j2.5L$ so $L = 3/2.5 = 1.2$ H

This represents an impedance of $j48\ \Omega$ at $\omega = 40$ rad/s and $j36\ \Omega$ at $\omega = 30$ rad/s

Then

$$-j5 = \frac{-j}{\omega C} = \begin{cases} -j0.3125\ \Omega,\ \omega = 40 \text{ rad/s} \\ -j0.417\ \Omega,\ \omega = 30 \text{ rad/s} \end{cases}$$

Then,

$$0.004\angle 0 = \frac{\mathbf{V}_{1a}}{j48} + \frac{\mathbf{V}_{1a} - \mathbf{V}_{2a}}{-j0.3125}$$

$$0 = \frac{\mathbf{V}_{2a}}{2} + \frac{\mathbf{V}_{2a} - \mathbf{V}_{1a}}{-j0.3125}$$

Solving,

$$\mathbf{V}_{1a} = 0.00814\angle -6.5°\text{ V}$$
$$\mathbf{V}_{2a} = 0.00804\angle 2.4°\text{ V}$$

Then, for the other frequency,

$$0 = \frac{\mathbf{V}_{1b}}{j36} + \frac{\mathbf{V}_{1b} - \mathbf{V}_{2b}}{-j0.417}$$

$$0.004\angle -90° = \frac{\mathbf{V}_{2b}}{2} + \frac{\mathbf{V}_{2b} - \mathbf{V}_{1b}}{-j0.417}$$

Solving,

$$\mathbf{V}_{1b} = 0.00808\angle -86.8°\text{ V}$$
$$\mathbf{V}_{2b} = 0.00799\angle -86.8°\text{ V}$$

64. Open circuit both sources, then remove the $2 - j\ \Omega$ impedance and look into the open terminals:

$$\mathbf{Z}_{Th} = j2 + 4\angle 10° = 4.77\angle 34.4°\ \Omega$$

Next, we re-activate the two current sources and compute the voltage across the open terminals:

$$\mathbf{V}_{oc} = \left(1.5\angle 24°\right)(j2) - (-2\angle 38°)(4\angle 10°) = 9.62\angle 64.6°\ \text{V}$$

Then $\mathbf{I}_1 = (\mathbf{V}_{oc})/[\mathbf{Z}_{th} + (2 - j)] = \dfrac{\mathbf{V}_{oc}}{\mathbf{Z}_{Th} + \left(2 - j\right)} = \dfrac{9.62\angle 64.6°}{4.77\angle 34.4° + 2 - j} = \boxed{1.56\angle 48.7°\ \text{A}}$

65. Starting with the left-hand source, we obtain a voltage source

$(1.5\angle 24°)(j2) = 3\angle 114°\ \text{V}$ in series with the impedance $j2\ \Omega$.

Moving to the right-hand source, we replace it along with the $4\angle 10°\ \Omega$ impedance by a voltage source
$(-2\angle 38°)(4\angle 10°) = 8\angle -132°\ \text{V}$ in series with the impedance $4\angle 10°\ \Omega$.

Both of these combinations appear in series with the $1 + j\ \Omega$ impedance, so

$$\mathbf{I} = (3\angle 114° - 8\angle -132°)(j2 + 1 + j + 4\angle 10°) = \boxed{1.56\angle 27.8°\ \text{A}}$$

66. (a)

$$\mathbf{Z}_{Th} = (12 - j34) \parallel j10 = \frac{(12 - j34)(j10)}{12 - j24} = \boxed{13.4\angle 82.9^\circ\ \Omega}$$

and

$$\mathbf{V}_{oc} = (22\angle 30^\circ)(13.4\angle 82.9^\circ) = \boxed{295\angle 113^\circ\ \text{V}}$$

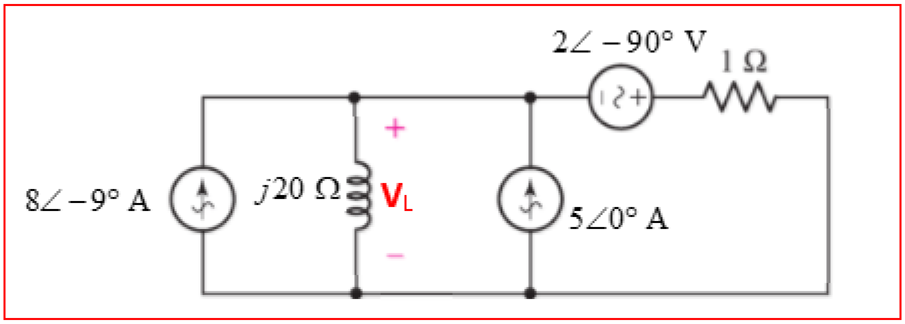
(b) Short-circuiting the two open terminals leads to (by inspection):

$\boxed{\mathbf{I}_N = 22\angle 30^\circ\ \text{A}}$; From above, $\boxed{\mathbf{Z}_{th} = 13.4\angle 82.9^\circ\ \Omega}$

(c) Utilize the Norton equivalent:

$$\mathbf{I} = (22\angle 30^\circ)\frac{13.4\angle 82.9^\circ}{13.4\angle 82.9^\circ + 7 - j2} = \boxed{0.942\angle 30.4^\circ\ \text{A}}$$

67. (a) Redrawn:



(b) Combining current sources, we may write (with the assistance of a quick source transformation):

$$8\angle -9° + 5\angle 0 = 12.96\angle -5.5° \text{ A}$$

$$12.96\angle -5.5° - \left(\frac{2\angle -90°}{1}\right) = 12.92\angle 3.3° \text{ A}$$

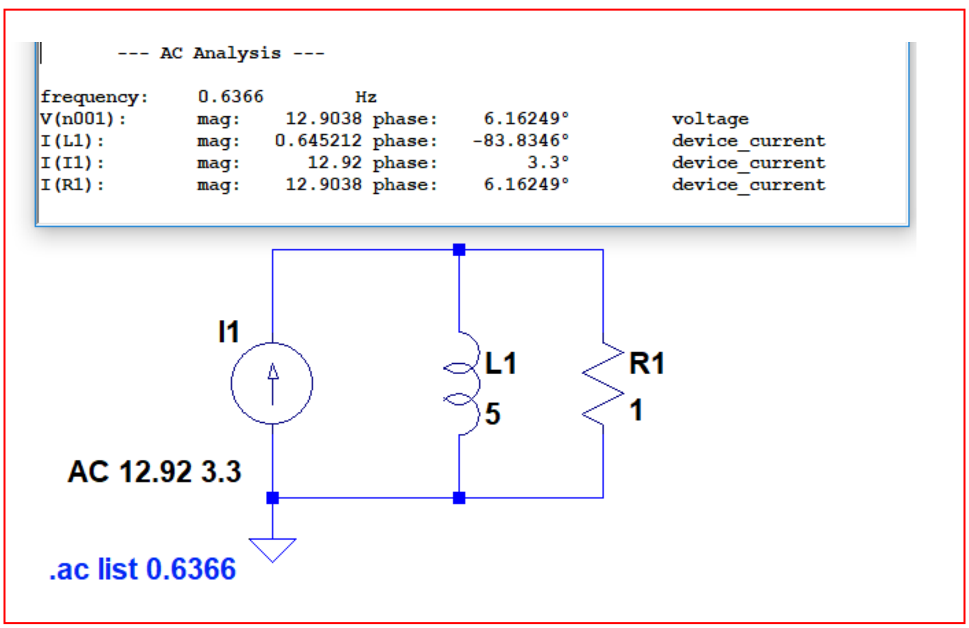
and hence,

by current division,

$$\mathbf{I}_L = 12.92\angle 3.3°\left(\frac{1}{1+j20}\right) = \boxed{0.645\angle -83.8° \text{ A}}$$

(c) $\mathbf{V}_L = \mathbf{I}_L(j20) = \boxed{12.9\angle 6.3° \text{ V}}$

(d)



68. Apply superposition, beginning with the $3\angle 0$ A source:

Nodal analysis yields

$$0 = \frac{\mathbf{V}_x}{j} + \frac{\mathbf{V}_x - \mathbf{V}_y}{-j} + \frac{\mathbf{V}_x}{1} \qquad [1]$$

$$-3\angle 0 = \frac{\mathbf{V}_y - \mathbf{V}_x}{-j} + \frac{\mathbf{V}_y}{2} \qquad [2]$$

Grouping terms,

$$\mathbf{V}_x - j\mathbf{V}_y = 0$$

$$-j\mathbf{V}_x + (0.5 + j)\mathbf{V}_y = -3$$

Solving,

$$\mathbf{V}_x = 1.664\angle -124° \text{ V}$$

$$\mathbf{V}_y = 1.664\angle 146° \text{ V}$$

For the $2.1\angle 0$ V source, we note that $(1) \,||\, (2 - j) = 0.707\angle -8.1°\ \Omega$.

Then, $\mathbf{V}_A = \dfrac{(2.1\angle 0)(j)}{j + 0.707\angle -8.1°} = 1.84\angle 37.9°$ V

$$\mathbf{V} = \mathbf{V}_A - 2.1\angle 0 = 1.3\angle 120° \text{ V}.$$

Finally,

$$\mathbf{V}_1 = 1.664\angle -124° - 1.3\angle 120° \text{ V} = 2.52\angle -96.3° \text{ V}$$

Returning to the time domain,

$$v_1(t) = 2.52\cos(20t - 96.3°) \text{ V}$$

69. We write two nodal equations:

$$5\angle 0 = \frac{\mathbf{V}_1}{-j3.83} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{1} \qquad [1]$$

$$0 = \frac{\mathbf{V}_2 - \mathbf{V}_1}{1} + \frac{\mathbf{V}_2}{-j2} + \frac{\mathbf{V}_2 - 110\angle 0}{5} \qquad [2]$$

Grouping terms,

$$(1 + j0.3)\mathbf{V}_1 - \mathbf{V}_2 = 5$$

$$-\mathbf{V}_1 + (1 + j0.5 + 0.2)\mathbf{V}_2 = \frac{110}{5}$$

Solving,

$\mathbf{V}_1 = 32.63\angle -81.6°$ V, or in the time domain, $v_1(t) = 32.63\cos(20t - 81.6°)$ V

$\mathbf{V}_2 = 32.27\angle -72.9°$ V, or in the time domain, $v_2(t) = 32.27\cos(20t - 72.9°)$ V

Therefore,

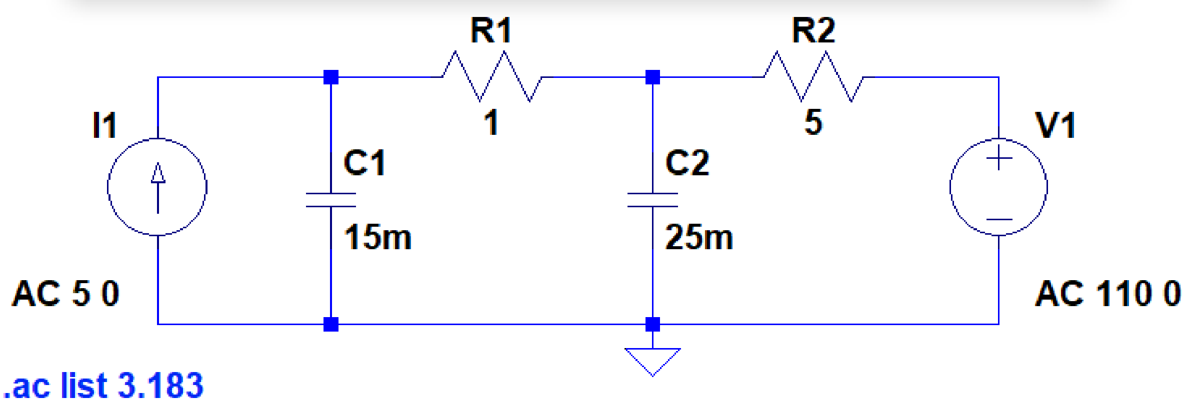
$$\mathbf{V}_{1\Omega} = \mathbf{V}_1 - \mathbf{V}_2 = 4.90\angle -163° \text{ V}$$

We proceed:

$$\mathrm{P}_{1\Omega} = \frac{\left(v_{1\Omega}(t)\right)^2}{1} = \boxed{24.01\cos^2(20t - 163°)\text{ W}}$$

```
       --- AC Analysis ---

frequency:    3.183        Hz
V(n001):      mag:   32.6335 phase:   -81.5699°        voltage
V(n002):      mag:   32.2662 phase:    -72.936°        voltage
V(n003):      mag:       110 phase:         0°         voltage
I(C2):        mag:   16.1326 phase:     17.064°        device_current
I(C1):        mag:   9.78976 phase:    8.43013°        device_current
I(I1):        mag:         5 phase:         0°         device_current
I(R2):        mag:   21.0315 phase:    17.0573°        device_current
I(R1):        mag:   4.89893 phase:    17.0355°        device_current
I(V1):        mag:   21.0315 phase:   -162.943°        device_current
```



70. We create the following impedances: 2 H becomes $j2\ \Omega$, and 1 F becomes $-j\ \Omega$
Apply a 1 A test source to the output terminals.

Then,

$$-0.25\mathbf{V}_L = \frac{\mathbf{V}_D}{-j} + \frac{(\mathbf{V}_D - \mathbf{V}_{test})}{j2} \qquad [1]$$

$$1 = \frac{(\mathbf{V}_{test} - \mathbf{V}_D)}{j2} \qquad [2]$$

$$\mathbf{V}_D - \mathbf{V}_{test} = \mathbf{V}_L \qquad [3]$$

So

$$\left(-0.25 - j - \frac{1}{j2}\right)\mathbf{V}_D + \left(0.25 + \frac{1}{j2}\right)\mathbf{V}_{test} = 0$$

$$-\mathbf{V}_D + \mathbf{V}_{test} = j2$$

Solving,

$\mathbf{V}_{test} = 1.12\angle 63.4°$ V and $\mathbf{Z}_{Th} = 1.12\angle 63.4°\ \Omega = 0.50 + j\ \Omega$

We may create such an impedance at 1 rad/s by inverting:

$$\frac{1}{\mathbf{Z}_{Th}} = 0.40 - j0.798$$

which can be constructed with $R = \frac{1}{0.4} =$ **2.5 Ω** in parallel with $\frac{1}{-j0.798} = j1.25\ \Omega$.

At 1 rad/s, that is a **1.25 H inductor.**

Shorting the output terminals,

$$-0.25\mathbf{V}_L + (\mathbf{V}_L - 1)(j) + \frac{\mathbf{V}_L}{j2} = 0$$

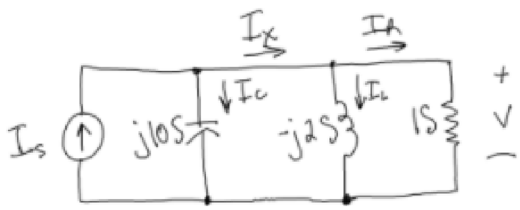
Solving,

$$\mathbf{V}_L = 36.7\angle -172.5°\ \text{V}$$

Thus,

$$\mathbf{I}_N = \mathbf{I}_{SC} = \frac{\mathbf{V}_L}{j2} = 18.4\angle 97.5°\ \text{A}$$

71.



a) $I_R = V(1) = (5 \angle 120^o)(1)$

$= (-2.5 + j4.33)(1)$

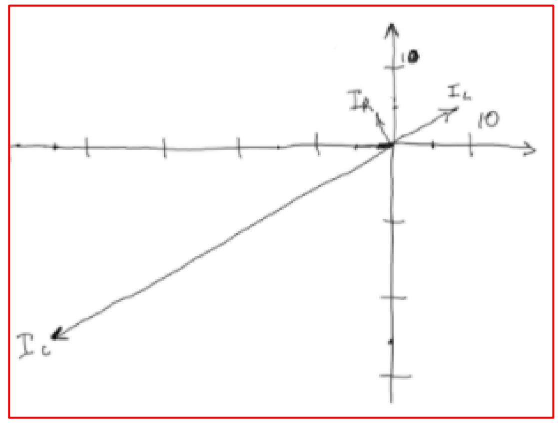
$= -2.5 + j4.33 \text{ A} = (5 \angle 120^o) \text{ A}$

$I_L = (5 \angle 120^o)(-j2)$

$= 8.66 + j5 \text{ A} = (10 \angle 30^o) \text{ A}$

$I_c = (5 \angle 120^o)(j10)$

$= -43.3 - j25 \text{ A} = (50 \angle -150^o) \text{ A}$

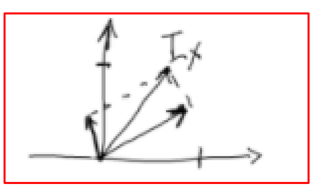


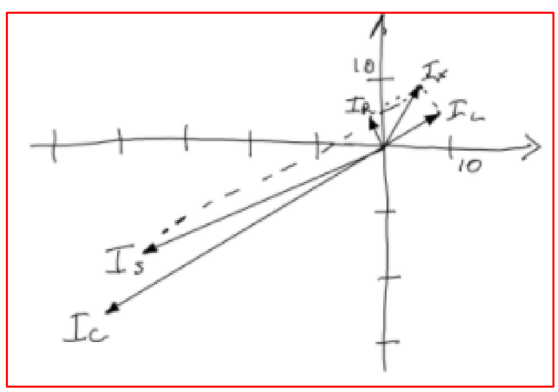
b) $I_x = I_R + I_L = -2.5 + j4.33 + 8.66 + j5$

$= 6.16 + j9.33 \text{ A} = (11.18 \angle 56.57^o) \text{ A}$

$I_s = I_x + I_c$

$= -37.14 - j15.67 \text{ A} = (40.31 \angle -157.13^o) \text{ A}$





72. $\mathbf{V}_1 = 100\angle 0^\circ$ V, $|\mathbf{V}_2| = 140$ V, $|\mathbf{V}_1 + \mathbf{V}_2| = 120$ V.
Let 50 V = 1 inch. From the sketch, for $\angle\mathbf{V}_2$ positive,
$\mathbf{V}_2 = 140\angle 122.5^\circ$. We may also have $\mathbf{V}_2 = 140\angle -122.5^\circ$ V

[Analytically: $|100 + 140\angle a| = 120$
so $|\ 100 + 140 \cos a + j140 \sin a\ | = 120$
Using the "Solve" routine of a scientific calculator,
$a = \pm 122.88^\circ$.]

73. (a) $\mathbf{I}_L = \dfrac{100}{j2.5 + \dfrac{-2}{2-j1}} = \dfrac{100(2-j1)}{2.5+j3} = \boxed{57.26\angle -76.76°\,(2.29in)}$

$\mathbf{I}_R = (57.26\angle -76.76°)\dfrac{-j1}{2-j1} = \boxed{25.61\angle -140.19°\,(1.02in)}$

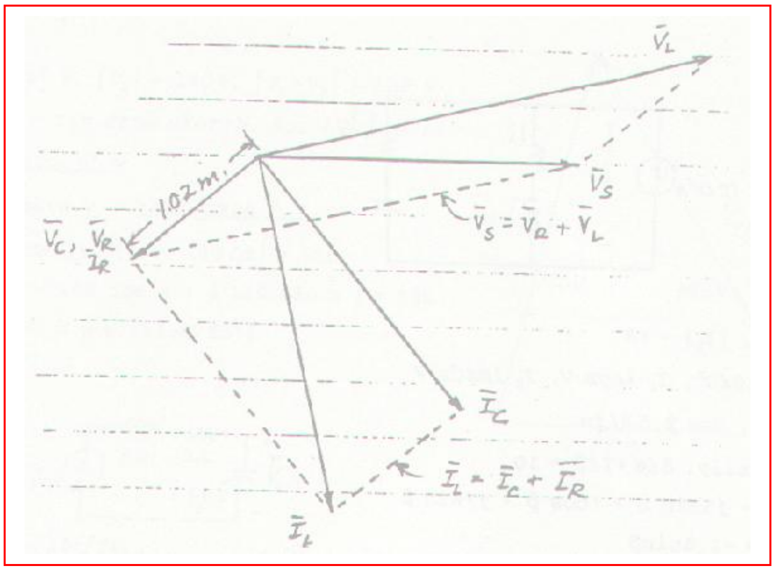
$\mathbf{I}_c = (57.26\angle -76.76°)\dfrac{2}{2-j1} = \boxed{51.21\angle -50.19°\,(2.05in)}$

$\mathbf{V}_L = 2.5 \times 57.26\angle 90° - 76.76° = \boxed{143.15\angle 13.24°\,(2.86in)}$

$\mathbf{V}_R = 2 \times 25.61\angle -140.19° = \boxed{51.22\angle -140.19°\,(1.02in)}$

$\mathbf{V}_c = \boxed{51.21\angle -140.19°\,(1.02in)}$

(b) Scales (inches) are noted above next to each answer.



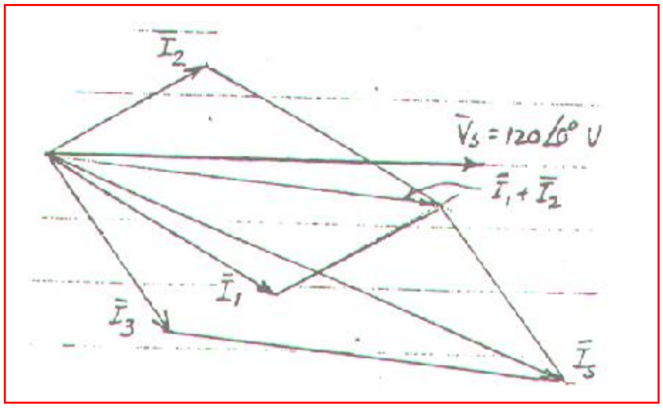
74. (a)

$$\mathbf{I}_1 = \frac{120}{40\angle 30^\circ} = 3\angle -30^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{120}{50 - j30} = 2.058\angle 30.96^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{120}{30 + j40} = 2.4\angle -53.13^\circ \text{ A}$$

(b)



(c) $\mathbf{I}_s = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3$

$= 6.265\angle -22.14^\circ \text{ A}$

75. $V_2 = I_c(j2) = (1 \angle 0^o)(j2)$

$= j2\ V = (2 \angle 90^o)\ V$

$I_c = \frac{V_R - V_2}{-j4}$

$(1 \angle 0^o) = \frac{V_R - (2 \angle 90)}{-j4}$

$V_R = (2 \angle -90^o)\ V = -j2\ V$

$I_s = I_R + I_c$

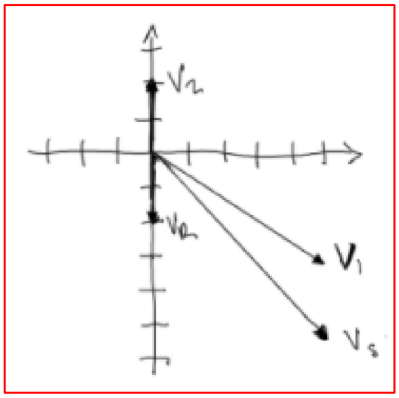
$I_s = \frac{V_R}{3} + I_c = (0.67 \angle -90^o) + (1 \angle 0^o) = 1 + j0.67\ A$

$= (1.2 \angle -33.7^o)\ A$

$V_1 = 5 * I_s = 5(1.2 \angle -33.7^o)$

$= (6.01 \angle -33.7^o)\ V = 5 - j3.33\ V$

a)



b) V_2 to V_1:

$\frac{V_2}{V_1} = -0.18 + j0.28$

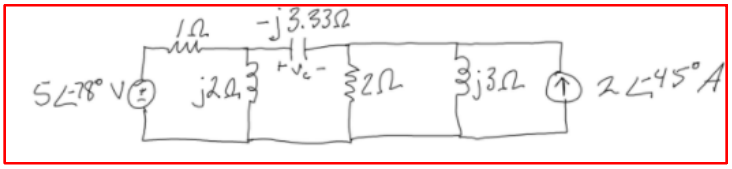
76. a) $X_{L1} = j\omega L = j(20)(100*10^{-3}) = j2\ \Omega$

$X_{L2} = j(20)(150*10^{-3}) = j3\ \Omega$

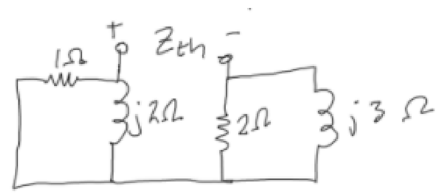
$X_c = \frac{-j}{\omega C} = \frac{-j}{(20)(15*10^{-3})} = -j3.33\ \Omega$

$V = (5 \angle -78^{o})$ V

$I = (2 \angle -45^{o})$ A



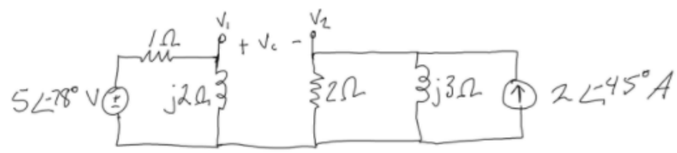
b) Short the voltage source and open the current source and make Z_{th} the impedance across the capacitor.



$Z_{th} = \frac{(1)(j2)}{1+j2} + \frac{(2)(j3)}{2+j3} = 0.8 + j0.4 + 1.38 + j0.92$

$= 2.18 + j1.32\ \Omega = (2.55 \angle 31.2^{o})\ \Omega$

Calculate the Thevenin equivalent voltage across the capacitor by open-circuiting the capacitor.



$V_1 = \frac{(1)(j2)}{1+j2}*(5 \angle -78^{o}) =$

$= (4.47 \angle -51.43^{o})\ V = 2.79 - j3.49\ V$

At node V_2:

$\frac{V_2}{2} + \frac{V_2}{j3} - (2 \angle -45^{o}) = 0$

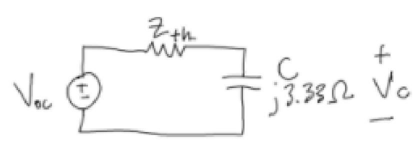
$V_2*(\frac{1}{2} + \frac{1}{j3}) = (2 \angle -45^{o})$

$V_2 = (3.33 \angle -11.34^{o}) = 3.20 - j0.64\ V$

$V_{oc} = V_1 - V_2 = 2.79 - j3.49 - (3.20 - j0.64)$

$= (-0.47 - j2.84)\ V = (2.88 \angle -99.46^{o})\ V$

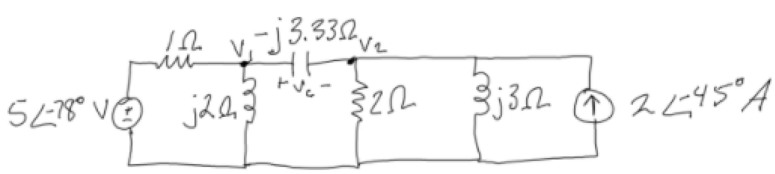
Thevenin equivalent circuit:



$V_c = \frac{-j3.33}{Z_{th} - j3.33} * V_{oc} = (3.23 \angle -146.74^{o})$

$V_c(t) = 3.23*\sin(20t - 146.74^{o})$ V

c)



Node V_1: $\frac{V_1-(5\angle -78)}{1} + \frac{V_1}{j2} + \frac{V_1-V_2}{-j3.33} = 0$

$V_c = V_1 - V_2$

$\frac{V_1-(5\angle -78)}{1} + \frac{V_1}{j2} + \frac{V_c}{-j3.33} = 0$

$V_1*(1 - j0.5) = (5\angle -78^o) + (0.97\angle 123.46^o)$

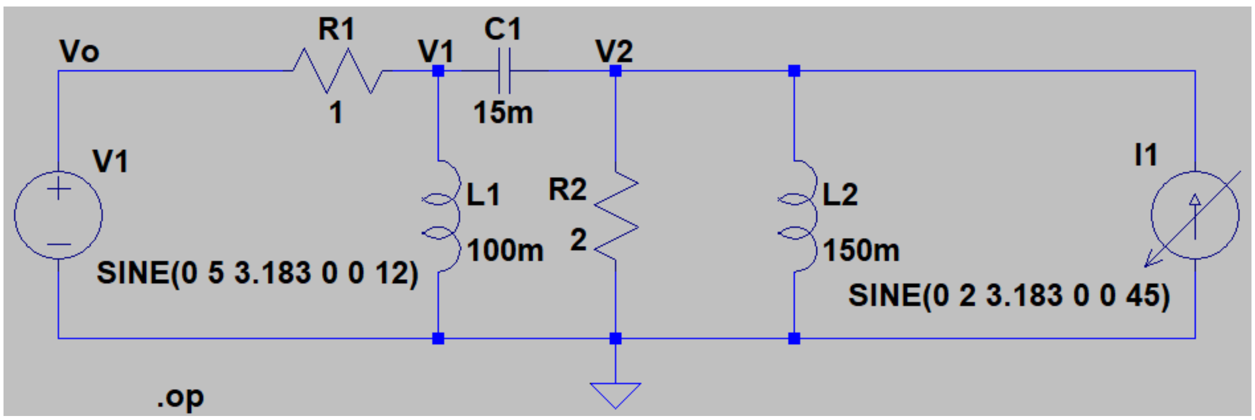
$V_1 = \frac{0.51+j4.08}{1-j0.5} = (2.04 - j3.06)$ V

$V_1 = (3.68\angle -56.34^o)$ V

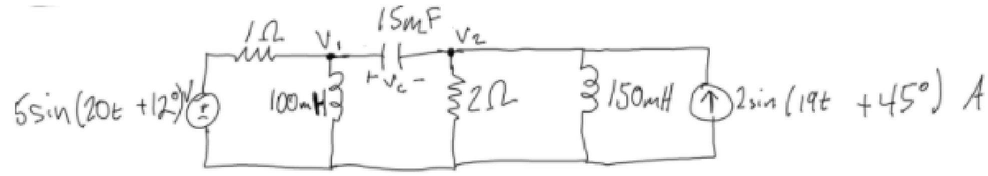
$I_c = \frac{(5\angle -78)-V_1}{1} = (5\angle -78^o) - (3.68\angle -56.34^o)$

$= (-1.0 - j1.83)$ A $= (2.08\angle -118.69^o)$ A

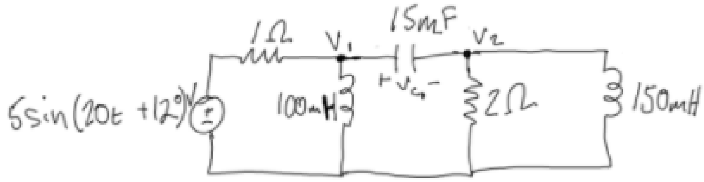
d)



77.



Superposition theorem:

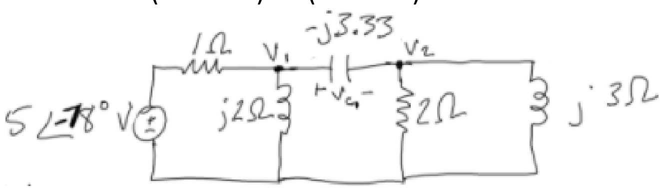


$X_{L1} = j\omega L = j(20)(100*10^{-3}) = j2\ \Omega$

$X_{L2} = j(20)(150*10^{-3}) = j3\ \Omega$

$X_c = \frac{-j}{\omega C} = \frac{-j}{(20)(15*10^{-3})} = -j3.33\ \Omega$

$5\sin(20t + 12) \rightarrow (5\angle -78^{\circ})$



Node V_1:

$\frac{V_1-(5\angle -78)}{1} + \frac{V_1}{j2} + \frac{V_1-V_2}{-j3.33} = 0$

$V_1*(1 + \frac{1}{j2} - \frac{1}{j3.33}) + V_2*\frac{1}{-j3.33} = (5\angle -78^{\circ})$

$V_1*(1 - j0.2) + V_2*(-j0.3) = 1.04 - j4.89$

Node 2:

$\frac{V_2-V_1}{-j3.33} + \frac{V_2}{2} + \frac{V_2}{j3} = 0$

$V_1*\frac{1}{j3.33} + V_2*(\frac{1}{j3} + \frac{1}{2} - \frac{1}{j3.33}) = 0$

$V_1*(-j0.3) = -V_2*(0.5 - j0.03)$

$V_1 = -V_2*(\frac{0.5 - j0.03}{-j0.3})$

$V_1 = (-0.1 - j1.67)*V_2$

$(-0.1 - j1.67)*V_2*(1 - j0.2) + V_2*(-j0.3) = 1.04 - j4.89$

$(-0.43 - j1.95)*V_2 = 1.04 - j4.89$

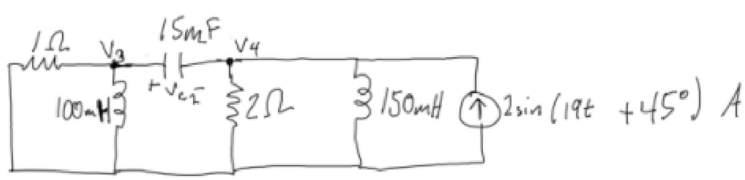
$V_2 = 2.28 + j1.04$ V

$V_1 = (4.19\angle -68.91^{\circ})$ V $= (1.51 - j3.91)$ V

$V_{c1} = V_1 - V_2$

$= (-0.77 - j4.96)$ V $= (5.02\angle -98.82^{\circ})$ V

Other side of circuit:

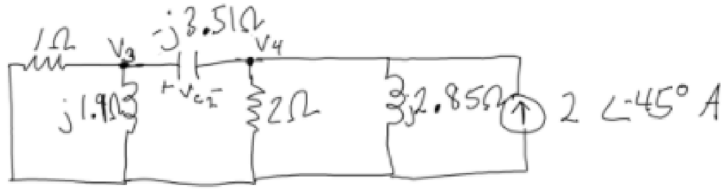


$X_{L1} = j\omega L = j(19)(100*10^{-3}) = j1.9\ \Omega$

$X_{L2} = j(19)(150*10^{-3}) = j2.85\ \Omega$

$X_c = \frac{-j}{\omega C} = \frac{-j}{(19)(15*10^{-3})}$ = -j3.51 Ω

2sin(19t +45°) → (2 ∠ -45°) A



Node V_3:

$\frac{V_3}{1} + \frac{V_3}{j1.9} + \frac{V_3 - V_4}{-j3.51} = 0$

V_3*(1 – j0.24) + V_4*(-j0.29) = 0

$V_4 = V_3*(\frac{1-j0.24}{j0.29})$ = V_3*(-0.85 – j3.51)

Node V_4:

$\frac{V_4 - V_3}{-j3.51} + \frac{V_4}{2} + \frac{V_4}{j2.85}$ = (2 ∠ -45°)

V_1*(-j0.29) + V_4*(0.5 – j0.065) = (1.41 – j1.41)

V_1*(-j0.29) + V_3*(-0.85 – j3.51)*(0.5 – j0.065) = (1.41 – j1.41)

V_3 = (0.43 + j0.85) V

V_4 = (2.62 – j2.23) V

$V_{c2} = V_3 - V_4$

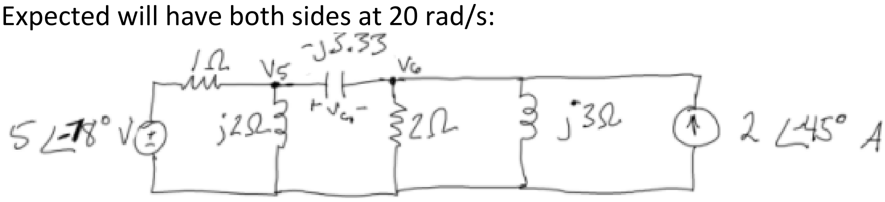
= (-2.20 – j3.09) V = (3.80 ∠ 125.42°) V

$V_c = V_{c1} + V_{c2}$

= (-2.97 – j1.87) V = (3.51 ∠ -147.85°) V

Actual: v_c(t) = 3.51cos(ωt – 147.85°) V

Expected will have both sides at 20 rad/s:



Node V_5:

$\frac{V_5 - (5\angle -78)}{1} + \frac{V_5}{j2} + \frac{V_5 - V_6}{-j3.33} = 0$

V_5*(1 – j0.2) + V_6*(-j0.3) = 1.04 – j4.89

$V_5 = \frac{(1.04 - j4.89) - V_6(-j0.3)}{1-j0.2}$ = (1.94 – j4.50) – (0.058 – j0.29)*V_6

Node 6:

$\frac{V_6 - V_5}{-j3.33} + \frac{V_6}{2} + \frac{V_6}{j3}$ = (2 ∠ -45°)

V_5*(-j0.3) + V_6*(0.5 – j0.03) = 1.41 – j1.41

(1.94 – j4.50) – (0.058 – j0.29)*V_6*(-j0.3) + V_6*(0.5 – j0.03) = 1.41 – j1.41

V_6 = (4.74 – j1.32) V

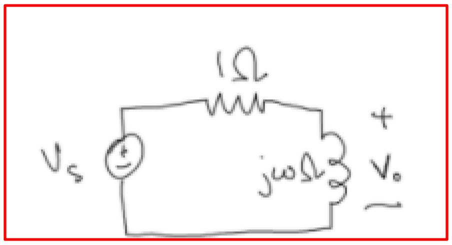
V_5 = (2.05 – j3.05) V

$V_c = V_5 - V_6$ =

= (-2.70 – j1.74) V = (3.23 ∠ -147.14°)

Expected: v_c(t) = 3.23cos(20t – 147.14°) V

78. a)

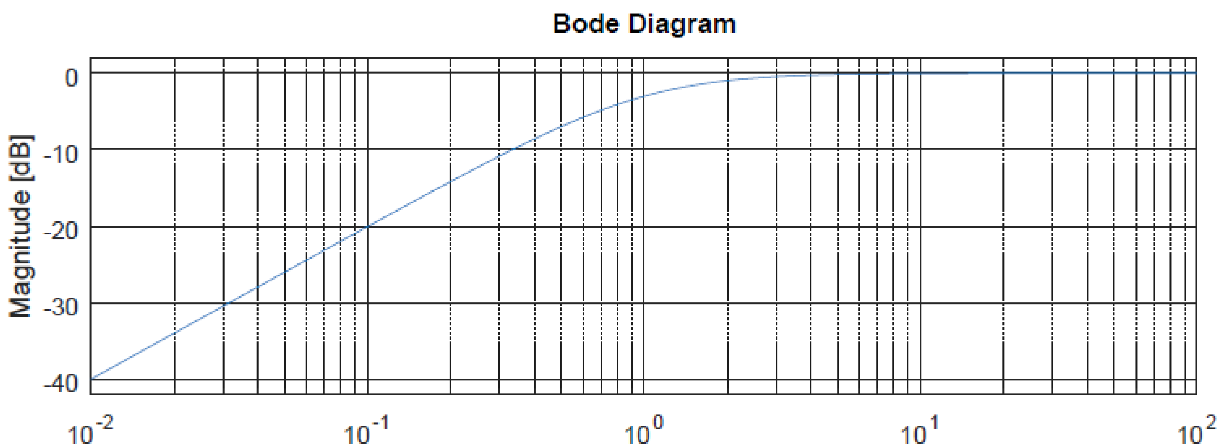


b) $V_o = \left(\frac{j\omega}{1+j\omega}\right)V_s$

$$\frac{V_o}{V_s} = \frac{j\omega}{1+j\omega}$$

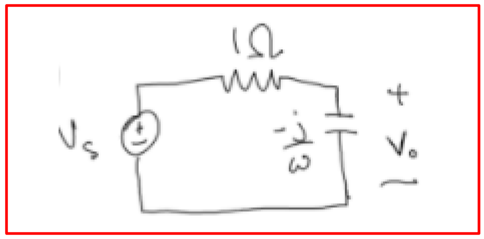
c) Using MATLAB:

```
Vo = [1 0];
Vs = [1 1];
w=0.01:0.01:100;
y=tf(Vo,Vs);
bode(y,{w})
grid on
```



d) Low frequencies are more effectively transferred to the output.

79. a)

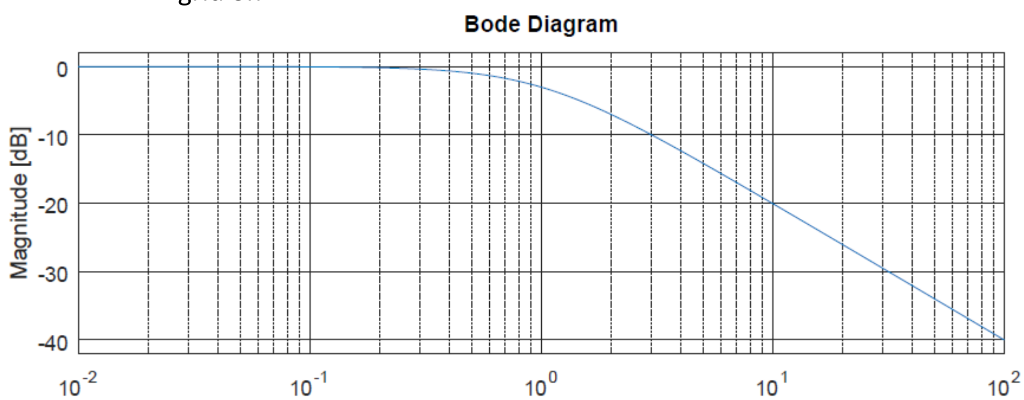


$V_o = \left(\frac{-j/\omega}{1-j/\omega}\right)V_s$

$\frac{V_o}{V_s} = \frac{-j}{\omega - j} = \frac{1}{1+j\omega}$

Using MATLAB:

```
Vo = [1];
Vs = [1 1];
w=0.01:0.01:100;
y=tf(Vo,Vs);
bode(y,{w})
grid on
```



Low frequencies are more effectively transferred to the output.

b) One possible solution (DESIGN PROBLEM):

Corner frequency: $f_c = \frac{1}{2\pi RC}$

$2*10^3 = \frac{1}{2\pi RC}$

$12.57*10^3 = \frac{1}{RC}$

Choose: R = 5 Ω. Then,

$C = \frac{1}{5(12.57*10^3)} =$ 15.91 μF

80. One possible solution (DESIGN PROBLEM):

Z = (0.5 ∠ 5.7°) Ω = 0.498 + j0.0497 Ω

F = 628 Hz

$\mathbf{Z} = R - jX_c = R - \frac{j}{\omega C}$

$= R - \frac{j}{2\pi f C}$

R = 0.50 Ω

$j0.0497 = \frac{j}{2\pi(628)C}$

C = 5.10 mF