1. Assuming the passive sign convention, $i_C = C\dfrac{dv_C}{dt}$.

(a) $\dfrac{dv_C}{dt} = 0 \therefore$ $\boxed{i = 0}$ (dc)

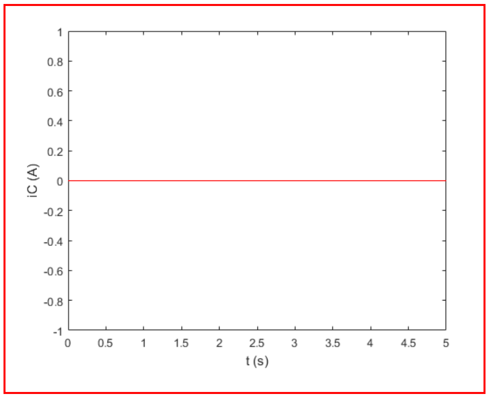
(b) $\dfrac{dv_C}{dt} = -10e^{-t} \therefore \boxed{i = (-10^{-9})e^{-t} \text{ A}}$

(c) $\dfrac{dv_C}{dt} = (2)(0.01)\cos(0.01t) \therefore \boxed{i = 2\times10^{-11}\cos\left(0.01t\right) \text{ A}}$

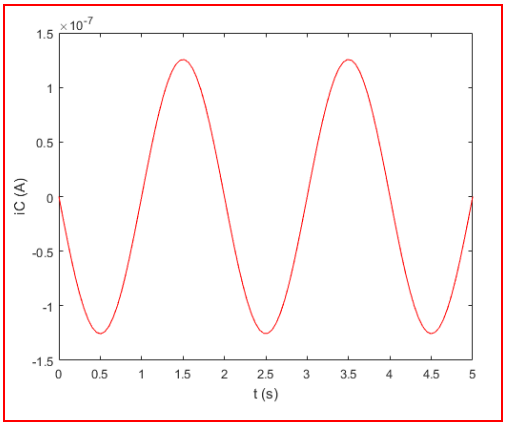
(d) $\dfrac{dv_C}{dt} = (2)(0.01)\cos(0.01t) \therefore \boxed{i = 2\times10^{-11}\cos(0.01t) \text{ A}}$

2. $i_C = C\dfrac{dv_C}{dt}$ where C = 10×10^{-9} F.

(a) $\dfrac{dv_C}{dt} = 0 \;\therefore\; i_C = 0$



(b) $\dfrac{dv_C}{dt} = -4\pi\sin t \;\therefore\; i_C = -40\times10^{-9}\pi\sin\pi t$ V

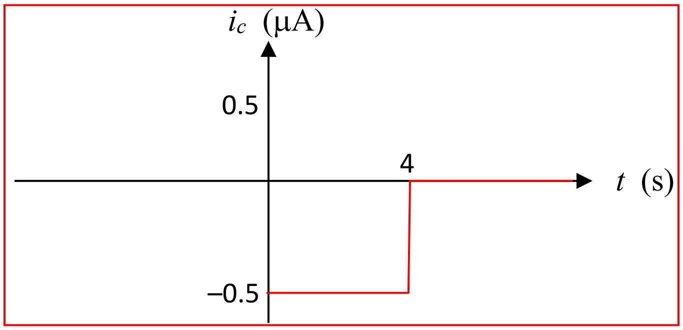


3. (a) C = 1 μF, assume passive sign convention.
For $t > 4$ s, $v = 1$ V therefore $i_C = 0$

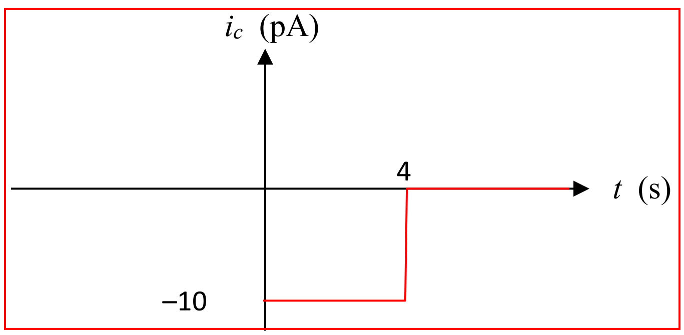
For all other times, $v = -\frac{1}{2}t + 3$

(a) C = 1 μF, assume passive sign convention.

$$i_C = C\frac{dv}{dt} = (10^{-6})\left(-\frac{1}{2}\right) = -0.5\times10^{-6}\text{ A} \quad \text{for } 0 \le t \le 4\text{ s}$$



(b) $i_C = (20\times10^{-9})\left(-\frac{1}{2}\right) = -10\text{ nA} \quad \text{for } 0 \le t \le 4\text{ s}$



4. Plate area = 2.5 mm × 2.5 mm = 6.25×10^{-6} m^2. D = 25×10^{-6} m.

$C = \dfrac{\varepsilon_r \varepsilon_0 A}{d}$. Thus,

(a) $C = \dfrac{(15)\left(8.854\times10^{-12}\right)\left(6.25\times10^{-6}\right)}{25\times10^{-6}} = \boxed{33.2 \text{ pF}}$

(b) $C = \dfrac{(1.5)\left(8.854\times10^{-12}\right)\left(6.25\times10^{-6}\right)}{25\times10^{-6}} = \boxed{3.32 \text{ pF}}$

(c) $C = \dfrac{(1)\left(8.854\times10^{-12}\right)\left(6.25\times10^{-6}\right)}{50\times10^{-6}} = \boxed{1.11 \text{ pF}}$

(d) $C = \dfrac{(1)\left(8.854\times10^{-12}\right)\left(12.5\times10^{-6}\right)}{25\times10^{-6}} = \boxed{2.21 \text{ pF}}$

5. $A = \pi r^2 = \pi\left(\frac{25\times10^{-3}}{2}\right)^2 = 4.91\times10^{-4} \quad \text{m}^2$

$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$. Therefore,

(a) $C = \frac{(1)\left(8.854\times10^{-12}\right)\left(4.91\times10^{-4}\right)}{0.1\times10^{-3}} = \boxed{43.5 \text{ pF}}$

(b) For mylar, $\varepsilon_r = 3.1$, so

$$C = \frac{(3.1)\left(8.854\times10^{-12}\right)\left(4.91\times10^{-4}\right)}{0.1\times10^{-3}} = \boxed{135 \text{ pF}}$$

(c) For SiO_2, $\varepsilon_r = 3.9$, so

$$C = \frac{(3.9)\left(8.854\times10^{-12}\right)\left(4.91\times10^{-4}\right)}{0.1\times10^{-3}} = \boxed{170 \text{ pF}}$$

6. **<DESIGN>** One possible solution:

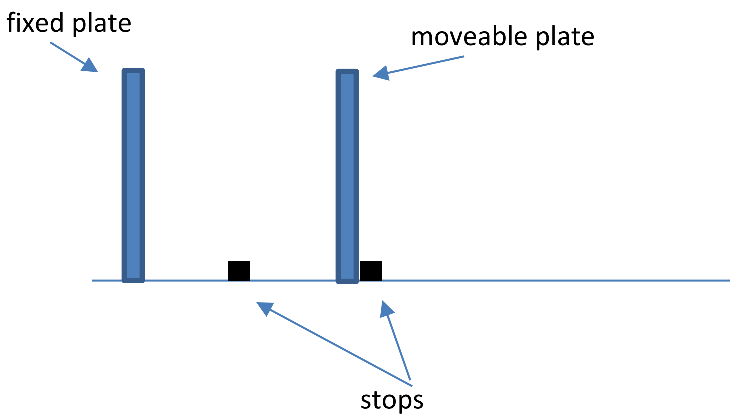
A standard AAA battery has a height of approximately 44.5 – 2(0.8) mm = 42.9×10^{-3} m.

Construct a parallel plate capacitor using two gold layers separated by a 1 μm thick layer of mylar (ε_r = 3.1).

Then, $C = \dfrac{(3.1)\left(8.854\times10^{-12}\right)A}{1\times10^{-6}} = \ 100\ \text{nF}$. Solving, we need each gold layer to have area A = 3.64×10^{-3} m^2. One side of each gold square is constrained by the height of the battery-sized container (42.9×10^{-3} m). Thus, the other side must be A/42.9×10^{-3} = 85 mm. This is much larger than the diameter (10.5 mm) of a AA battery, so we gently roll the structure along the length after first contacting what will be the inner gold foil.

7. <**DESIGN**> One possible solution:

We construct something as sketched here:



Our design has two 1mm thick metal plates facing each other, each with area A. One is fixed, and the other can move along the x axis. Stops are placed to restrict the maximum plate spacing to 10 mm. If the left side of the fixed plate is at x = 0, the left stop is at x = d, and the right stop is at 12 mm.

The dielectric layer is simply air, so $\varepsilon_r = 1$. Then

$$C = \frac{(1)\left(8.854\times10^{-12}\right)A}{10\times10^{-3}} = \quad 100 \text{ pF} \quad \text{which we can solve for A = 0.113 m}^2.$$

Taking the square root, each square plate must be 336 mm tall and 336 mm long.

The location of the left stop can now be determined:

$$C = \frac{(1)\left(8.854\times10^{-12}\right)(0.113)}{d} = \quad 200 \text{ pF} \quad \text{or d = 5 mm.}$$

8. **<DESIGN>** One possible solution:

$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$.

$$\frac{(\varepsilon_r)\left(8.854\times10^{-12}\right)A}{d_1} = 250 \text{ pF} \quad [1]$$

$$\frac{(\varepsilon_r)\left(8.854\times10^{-12}\right)A}{d_2} = 500 \text{ pF} \quad [2]$$

Divide Eq. [1] by Eq. [2] to yield $\frac{1}{2} = \frac{\frac{1}{d_1}}{\frac{1}{d_2}} = \frac{d_2}{d_1}$.

Also, we have the constraint that $d_1 - d_2 = 0.1\times10^{-3}$ m.

Solving together, $d_1 = 2\times10^{-4}$ m and $d_2 = 1\times10^{-4}$ m.

Use silicone as the spacer ($\varepsilon_r = 2.9$). Construct the capacitor using two squares of copper foil, each 1 mm thick, having area A. Coat one square on one side with a 2×10^{-4} m thick layer of silicone; place the other foil square directly on top of the silicone layer.

Now we can solve for the needed area of each piece of foil:

$\frac{(2.9)\left(8.854\times10^{-12}\right)A}{2\times10^{-4}} = 500 \times10^{-12}$ or $A = 9.74\times10^{-4}$ m^2. Hence each foil piece must measure 31.2 mm x 31.2 mm.

9. Taking care to convert all distance units to meters,

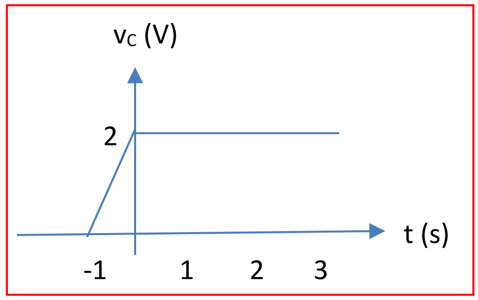
$$C = \frac{(11.8)(8.854\times10^{-12})(2\times10^{-12})}{W}$$

$$W = \left[\frac{(2)(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(5\times10^{24})}\left(0.62 - V_A\right)\right]^{\frac{1}{2}}$$

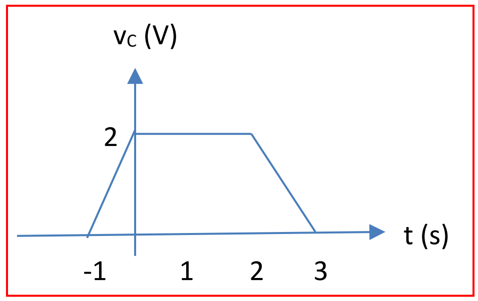
V_A	W (m)	C
-1 V	2.06×10^{-8}	10.2 fF
-3 V	3.07×10^{-8}	6.81 fF
-10 V	5.23×10^{-8}	4.00 aF

10. We note that $v_C = \frac{1}{C}\int i_C dt$ where C = 2 F.

(a) (1/2)(2)(1) = 4/2 V = 2 V.

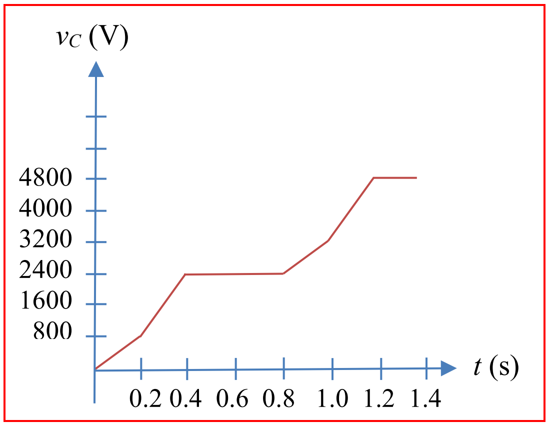


(b) (1/2)(2)(1) = 2 V



11. C = 1 mF. $i = C\dfrac{dv}{dt}$ so $v = \dfrac{1}{C}\int i dt$.

(a) $(10^3)(4)(0.2) = 800$ V
$(10^3)(8)(0.2) = 1600$ V
Then this repeats, and adds to what has been previously computed.



(b) $v(0.2)$ = 800 V

$v(0.6)$ = 2400 V

$v(1.2)$ = 4800 V

12. Energy stored is given by $w_C = \frac{1}{2}C\left(v_C\right)^2$.

(a) $w_C = \frac{1}{2}(1.4)\left(8\right)^2 = \boxed{44.8\text{ J}}$

(b) $w_C = \frac{1}{2}(22\times10^{-12})\left(0.8\right)^2 = \boxed{7.04\text{ pJ}}$

(c) $w_C = \frac{1}{2}(18\times10^{-9})\left\{\left(12\right)^2 - (2)^2\right\} + 295\times10^{-9} = \boxed{1.56\ \mu\text{J}}$

13. C = 150 pF

$$v_C(t) = \begin{cases} 12 \text{ V}, t < 0 \\ 12e^{-2t} \text{ V}, t \geq 0 \end{cases}. \text{ Energy} = \frac{1}{2}C[v_C]^2.$$

(a) t = 0: $\frac{1}{2}C[v_C]^2 = \frac{1}{2}(150\times10^{-12})[12]^2 = \boxed{10.8 \text{ nJ}}$

(b) t = 0.2 s: $\frac{1}{2}(150\times10^{-12})\{(8.04)^2 - (12)^2\} + 1.08\times10^{-8} = \boxed{4.85 \text{ nJ}}$

(c) t = 0.5 s: $v_C(0.5) = 4.41 \text{ V} \therefore \frac{1}{2}C[v_C]^2 = \frac{1}{2}(150\times10^{-12})[4.41]^2 = \boxed{1.5 \text{ nJ}}$

(d) t = 1 s: $v_C(1) = 1.62 \text{ V} \therefore \frac{1}{2}C[v_C]^2 = \frac{1}{2}(150\times10^{-12})[1.62]^2 = \boxed{192 \text{ pJ}}$

14. (a)

$$v_C = 1.2\frac{22}{40+22} = \boxed{0.426\ \text{V}}$$

$$P_{40} = i_{40}^2(40) = \left(\frac{1.2}{40+22}\right)^2(40) = \boxed{15\ \text{mW}}$$

(b) $\boxed{v_C = 0}$

$i_{40} = 0$ so $\boxed{P_{40} = 0}$

15. (a) no current through 5 Ω resistor so

$$-v_C = (4.5\times10^{-9})(13)\frac{7}{13+10+7} = 13.65\times10^{-9}\text{ V} \quad \text{so } \boxed{v_C = -13.65\text{ nV}}$$

(b) v_C = voltage across current source so

$$(4.5)[10+7\,||\,18] = \boxed{67.7\text{ nV}}$$

16. **<DESIGN>** One possible solution:

From Table 2.4, this wire has a cross-sectional area of 0.0804 mm^2.

$L = \mu N^2 \frac{A}{s}$ where here $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m due to air as filler.

Solving for the length *s* of our helix,

$$s = \frac{\mu_0 N^2 A}{L} = \frac{4\pi \times 10^{-7} N^2 A}{30 \times 10^{-9}} = 41.9 N^2 A$$

Select a coil radius of 2.5 mm, which is larger than our wire diameter. Select N2 = 20. Solving,

s = 329 mm (which is a bit over 1 ft long).

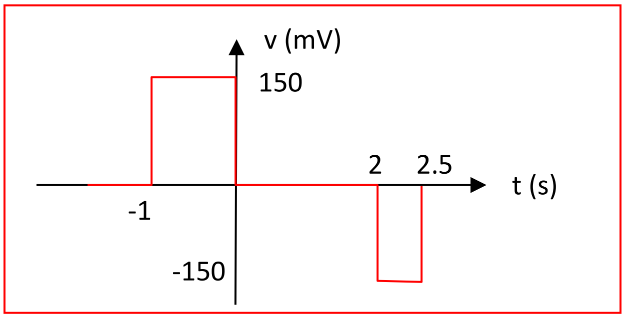
17. L = 75 mH.

$$v = L\frac{di}{dt}.$$

Voltage can change instantaneously but current cannot (or voltage becomes infinite).

From t = -1 to 0, i = 2(t + 1) so slope = +2. From t = 2 to 2.5, slope = -2.

(a)

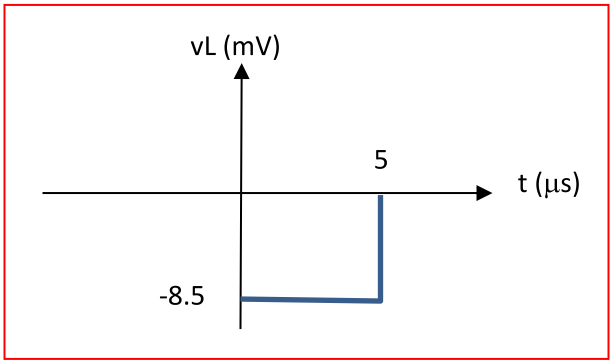


(b) t = 1, 2.9 s, 3.1 s

$v(1) = 0$; $v(2.9) = 0$; $v(3.1) = 0$;

18. L = 17 nH

Slope = $\frac{2-4}{5-1}\times 10^6 = -0.5\times 10^6$. $v_L = (17\times 10^{-9})(-0.5\times 10^6) = -8.5$ mV



19. $v = L\dfrac{di}{dt} = (4.0\times10^{-3})\dfrac{di}{dt}$

(a) $\dfrac{di_L}{dt} = 0 \therefore \boxed{v = 0}$

(b) $\dfrac{di_L}{dt} = (3)(6)\cos 6t \therefore \quad v = (4\times10^{-3})\cos 6t \text{ V} = \boxed{0.072\cos 6t \text{ V}}$

(c)

$$\frac{di_L}{dt} = (-115)(\sqrt{2})(100\pi)\sin\left(100\pi t - 9^o\right) \therefore$$

$$v = (4\times10^{-3})(-115)(\sqrt{2})(100\pi)\sin\left(100\pi t - 9^o\right)$$

$$= \boxed{-204\sin\left(100\pi t - 9^o\right) \text{ V}}$$

(d) $\dfrac{di_L}{dt} = -15e^{-t} \therefore v = -(4\times10^{-3})(15)e^{-t} = \boxed{-60\times10^{-3}e^{-t} \text{ V}}$

(e) $\dfrac{di_L}{dt} = t\left(-10e^{-10t}\right) + e^{-10t} \therefore v = \boxed{(4\times10^{-3})(1-10t)e^{-10t} \text{ V}}$

20. L = 8 pH, $v = L\dfrac{di}{dt} = (8\times10^{-12})\dfrac{di_L}{dt}$.

(a) $\dfrac{di_L}{dt} = 0 \therefore \boxed{v = 0}$

(b) $\dfrac{di_L}{dt} = 0 \therefore \boxed{v = 0}$

(c) $\dfrac{di_L}{dt} = 0 \therefore \boxed{v = 0}$

(d) $\dfrac{di_L}{dt} = -4e^{-t} \therefore \boxed{v = -16\times10^{-3}e^{-t} \text{ V}}$

(e) $\dfrac{di_L}{dt} = -te^{-t} + e^{-t} \therefore \boxed{v = 4\times10^{-3}(1-t)e^{-t} \text{ V}}$

21. is = 1 mA, vs = 2 V

(a) vL = 0; iL = is = 1 mA

(b) 14 kΩ resistor is irrelevant here. vL = 0; iL = is = 1mA

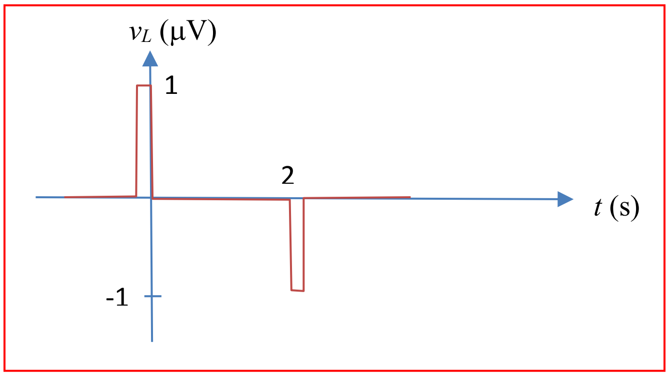
(c) vL = 0; iL = vs/4.7×10^3 = 426 μA

(d) vL = 0; iL = vs/14×10^3 = 143 mA

22. L = 200 nH. $v = L\frac{di}{dt} = (200\times10^{-9})\frac{di_L}{dt}$

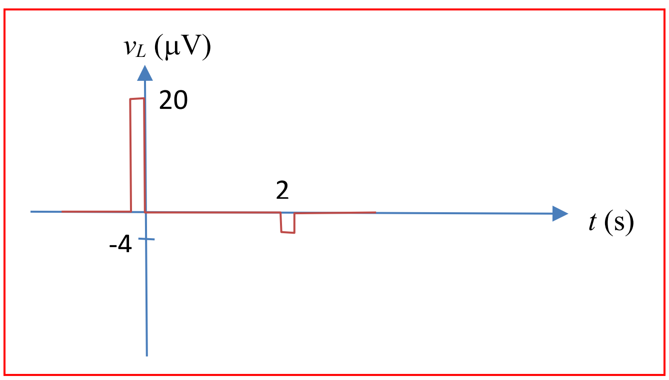
Since the current has a linear slope, $v = \frac{\Delta i_L}{\Delta t}$.

(a) Δt = 200 ms. The first pulse then results in $v = 200\times10^{-9}\frac{1}{200\times10^{-3}} = 1$ μV. The second pulse results in the same voltage but with opposite sign.



(b) The first pulse results in $v = 200\times10^{-9}\frac{1}{10\times10^{-3}} = 20$ μV. The second pulse results in

$v = 200\times10^{-9}\frac{1}{50\times10^{-3}} = 4$ μV.



23. $v_L = L\dfrac{di_L}{dt}$. L = 1 H.

Between -2 and 1 s, slope = (2 – 0)/(-2 – 0) = -1

Between 1 and 3 s, slope = -1/2

For t > 3 s, slope = -2/-3 = 2/3

(a) v(-1) = (1)(-1) = -1 V
(b) v(0) = (1)(-1) = -1 V
(c) v(1.5) = (1)(-1/2) = -0.5 V
(d) v(2.5) = (1)(-1/2) = -0.5 V
(e) v(4) = (1)(2/3) = 0.67 V
(f) v(5) = 0.67 V

24. $i_L = \frac{1}{6\times10^{-3}}\int v_L dt$ where L = 6 mH.

(a) Any dc current (including 0 A) will result in a zero voltage across the inductor.

(b) $i_L = \frac{1}{6\times10^{-3}}\int 100\sin 120\pi t\ dt = \frac{100}{(6\times10^{-3})(120\pi)}(-\cos 120\pi t)$

$= -44.2\cos 120\pi t$ A

25. $v_L = 4t, \quad \text{and } i_L(-0.1) = 100\ \mu\text{A}$

$$i_L = \frac{1}{L}\int_{-0.1}^{t} 4t'dt' + i_L(-0.1) = \frac{4}{2}\left[t^2 - (-0.1)^2\right] + 100\times10^{-6}$$

(a) At t = 0,

$$i_L = \frac{4}{2}\left[-\left(-0.1^2\right)^2\right] + 100\times10^{-6} = \boxed{-19.9\ \text{mA}}$$

(b) At t = 1.5 ms,

$$i_L = \frac{4}{2}\left[(0.0015)^2 - \left(-0.1^2\right)^2\right] + 100\times10^{-6} = \boxed{-19.9\ \text{mA}}$$

(c) At t = 45 ms,

$$i_L = \frac{4}{2}\left[(0.045)^2 - \left(-0.1^2\right)^2\right] + 100\times10^{-6} = \boxed{-15.9\ \text{mA}}$$

26. L = 1 nH. $w_L = \frac{1}{2}Li^2$.

(a) $i = 0$ therefore $\boxed{w_L = 0 \text{ J}}$

(b) $i = 1 \text{ mA} \therefore w_L = \frac{1}{2}(10^{-9})(10^{-3})^2 = \boxed{5\times10^{-16} \text{ J}}$

(c) $i = 20\text{A} \therefore w_L = \frac{1}{2}(10^{-9})(20)^2 = \boxed{2\times10^{-7} \text{ J}}$

(d) $i = 5\sin 6t \text{ mA} \therefore w_L = \frac{1}{2}(10^{-9})(5\times10^{-3}\sin 6t)^2 = \boxed{1.25\times10^{-14}\sin^2 6t \text{ J}}$

27. L = 33 mH, t = 1 ms. $w = 0.5Li^2$

(a) $i = 7\text{ A} \therefore w_L = \frac{1}{2}(33\times10^{-3})(7)^2 =$ **809 mJ**

(b) $i = 3 - 9e^{-10^3(10^{-3})}\text{ A} \therefore w_L = \frac{1}{2}(33\times10^{-3})(-0.311)^2 =$ **1.60 mJ**

28. (a) A source transformation results in a (10/16) mA current source in parallel with 16 kΩ. Then, by current division,

$$i_x = \left(\frac{10}{16}\right)\frac{\frac{1}{7}}{\frac{1}{10}+\frac{1}{4.7}+\frac{1}{4.7}+\frac{1}{7}} \text{ mA} = \boxed{134 \text{ mA}}$$

(b) We can neglect the left-hand portion of the circuit and invoke current division:

$$i_x = 10\frac{2}{2+5} = \boxed{2.86 \text{ A}}$$

29. (a) $V/27 + V/30 + (V-1)/20 = -5$

Solving, V = –41.12 V

By voltage division, Vx = (12/27)(–41.95) = **–18.3 V**

(b) $V/27 + V/20 + (V-1)/20 = -5$

Solving, V = –36.12 V

By voltage division, Vx = (12/27)(–36.12) = **–16.1 V**

(c) $V/27 + (V-1)/20 = -5$

V = –56.87 V

By voltage division, Vx = (12/27)(–56.87) = **–25.28 V**

(d) Same as case (b): Vx = **–25.28 V**

30. (a) Our equivalent is found as:

$$V_{Th} = 4\frac{47}{10+47} = \boxed{3.3\ \text{V}}$$

$$R_{Th} = 10\ \text{k}\Omega\ ||\ 47\ \text{k}\Omega = \boxed{8.25\ \text{k}\Omega}$$

(c) The 47 kΩ resistor is shorted out, therefore $\boxed{P_{47k} = 0}$

and $P_{10k} = \dfrac{4^2}{10\times10^3} = \boxed{1.6\ \text{mW}}$

(d) $w_L = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(0.05)\left(\dfrac{4}{10\times10^3}\right)^2 = \boxed{4\ \text{nJ}}$

31. Left leg: 1 F in series with 1 F = (1)(1)/(1+1) = 0.5 F. This appears in parallel with 1 F, so the group can be replaced with 1 + 0.5 = 1.5 F. The right leg is identical. Thus, we are left with 1.5 F in series with (1+1 = 2 F) in series with 1.5 F, or

$$C_{eq}^{-1} = \frac{1}{1.5} + \frac{1}{2} + \frac{1}{1.5} = \boxed{545.4 \text{ mF}}$$

32. Each Δ reduces to (L + L) || L = (2L)(L)/(2L + L) = 2L/3 H.

This leaves us with (2L/3) + L + (2L/3) = 2.33L.

33. **<DESIGN>** Two possible solutions:

(a) Connect one inductor in series with a parallel connection of 4 inductors.
Then, Leq = 1 + 1||1||1||1 = 1 + 0.25 = 1.25 nH.

(b) Create a string by connecting two inductors in series with a parallel combination of two inductors. Then connect two such strings in parallel.

Leq = 0.5*(1 + 1 + 1 || 1) = 0.5*(2.5) = 1.25 nH

34. We have 2 F in series with 1 F in series with 2 F, which equates to

$$\left(\frac{1}{2}+\frac{1}{1}+\frac{1}{2}\right)^{-1} = 0.5 \text{ F}$$

Next we have 8 F in series with 5 F, which equates to (8)(5)/(8+5) = 3.08 F.

These two combinations appear in parallel, and equate to 0.5 + 3.08 = 3.58 F.

Now we have 4 F in series with 3.58 F in series with 1 F, or

$$\left(\frac{1}{4}+\frac{1}{3.58}+\frac{1}{1}\right)^{-1} = 0.654 \text{ F}$$

Continuing the same process, we now have 5 F in parallel with 0.654 F, or 5.654 F.

Finally, this appears in series with 7 F, so that

$$C_{eq} = \left(\frac{1}{5.654}+\frac{1}{7}\right)^{-1} = \boxed{3.13 \text{ F}}$$

35. The left leg can be reduced to
$(1/7 + 1/22)^{-1} + 4 = 9.310$ F

On the right leg, we neglect the 2 F, and then combine as
$5 + (1/12 + 1/1)^{-1} = 5.923$ F

Thus, $C_{eq} = (1/9.310 + 1/5.923)^{-1} =$ **3.62 F**

36. The left network reduced to [12||10 + 7] || 4 = [(12)(10)/(12 + 10) + 7] ||4

or $\frac{[12.455](4)}{12.455+4} = 3.03 \text{ H}$.

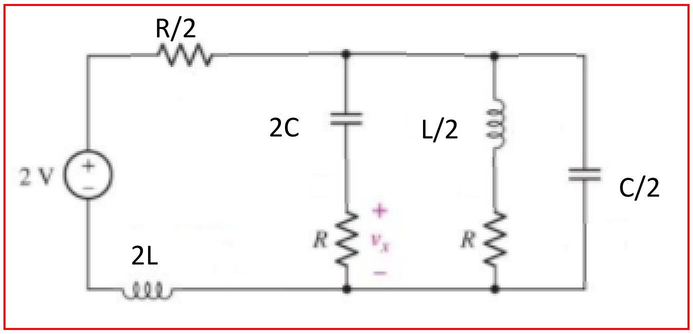
On the right network, we ignore the 2 H, and form the combination (12 + 1) || 5

or $\frac{(13)(5)}{13+5} = 3.61 \text{ H}$

Thus, L_{eq} = 3.03 + 3.61 = **6.64 H**

37. We note that R||R = R/2, C || C = 2C, L ||L = L/2, L + L = 2L, and two capacitors in series equate to C/2.

Thus, our network can be reduced to:



38. (a) We start by naming each resistor as "R" so that we can create an expression and substitute in the value of 10 Ω at the end.

Noting that R||R||R = 3R,

The network can be reduced to

$$3R \,||\left[\frac{R}{2}+\left(R+\frac{R}{3}\right)\right]=\left(\frac{(3R)(11R/6)}{3R+11R/6}\right)=\frac{(30)(110/6)}{30+(110/6)}=\boxed{11.4\ \Omega}$$

(b) Inductor combinations work the same way as resistors, so Leq = $\boxed{11.4\text{ H.}}$

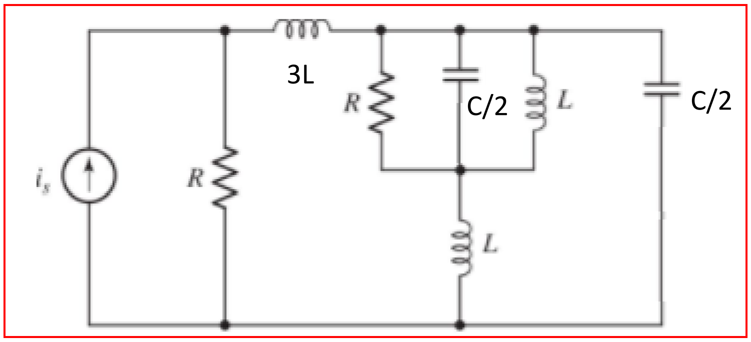
(c) $$C_{eq}=\frac{C}{3}+\left[\frac{1}{2C}+\frac{1}{C}+\frac{1}{3C}\right]^{-1}=\frac{10}{3}+\left[\frac{1}{20}+\frac{1}{10}+\frac{1}{30}\right]^{-1}=\boxed{8.79\text{ F}}$$

39. All inductances are in parallel, so $1/L_{eq} = (1/1 + ½ + 1/1 + 1/7 + ½ + ¼)\times10^9$

Thus, $L_{eq} = 295$ pH

40. Two capacitors of value C in series can be replaced by C/2.

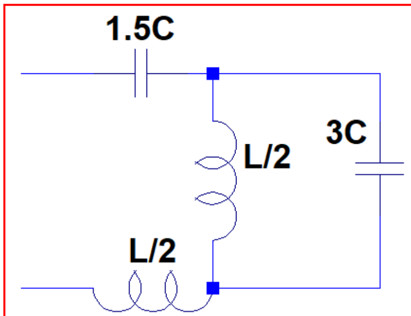
Three inductors of value L in series can be replaced by 3L. Thus, our circuit can be reduced to:



41. Temporarily assign capacitors value 'C' and inductors value 'L.' Then at the far right we can neglect the two inductors in parallel. Both vertical inductors are actually in parallel, to be replaced by L*L/(L+L) = L/2. We have three sets of two capacitors in series, each of which can be replaced with C*C/(C+C) = C/2. The middle group of vertical capacitors can be replaced with

$$C + C/2 + C/2 + C = 3\ C.$$

Finally, the upper left combination of three capacitors can be replaced with C + C/2 = 1.5C. Our equivalent circuit is then



Since L = 2 nH and C = 2 nF, 1.5C = 3 nF, 3C = 6 nF, and L/2 = 1 nH.

42. $L_{eq} = L_1 + L_2||L_3 + L_4||L_5||L_6$

or

$$L_{eq} = L_1 + \frac{L_2 L_3}{L_2 + L_3} + \left(\frac{1}{L_4} + \frac{1}{L_5} + \frac{1}{L_6}\right)^{-1}$$

$$= 1 + \frac{(2)(2)}{(2)+(2)} + \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right)^{-1} = 1 + 1 + 1 = \boxed{3\text{ H}}$$

43. At the far right, we have 10 F in series with 10 F, which can be replaced with 10/2 = 5 F. We then have 10 F || 5 F = 15 F. This is in series with 10 F. That combination is equivalent to (15)(10)/(15+10) = 6 F. This is in parallel with 10 F, for an equivalent of 16 F.

Finally, the equivalent of 16 F, which represents all but the bottom left capacitor, is in series with that 10 F, for an equivalent of 16(10)/(16+10) = 6.15 F.

44. Starting on the far right, we see 10 + 10 = 20 H. This is in parallel with 10 H, so the combination may be replaced by 20(10)/(20 + 10) = 6.67 H.

This is in series with a 10 H inductor, so the combination may be replaced with 10 + 6.67 = 16.67 H. This is in parallel with 10 H, for an equivalent of (10)(16.67)/(10 + 16.67) = 2.5 H.

Finally, the equivalent inductance of 2.5 H is in series with the bottom left 10 H inductor, so

$L_{eq} = 10 + 2.5 = \boxed{12.5 \text{ H}}$

45. $$0 = C_1 \frac{d(v_1 - v_3)}{dt} + C_2 \frac{dv_1}{dt} + \frac{v_1 - v_2}{R}$$

or $$(C_1 + C_2)\frac{dv_1}{dt} + \frac{v_1}{R} - \frac{v_2}{R} - C_1 \frac{dv_3}{dt} = 0 \qquad [1]$$

Next, $-is = \frac{v_2}{R} + \frac{v_2 - v_1}{R} + \frac{1}{L}\int_{t_0}^{t}(v_2 - v_5)\,dt' - i_L(0^-)$ or

$$-\frac{v_1}{R} + 2\frac{v_2}{R} + \frac{1}{L}\int_{t_0}^{t} v_2\,dt' = i_L(0^-) - is + \frac{1}{L}\int_{t_0}^{t} v_s\,dt' \quad [2]$$

By inspection, $v_3 = v_s$.

46. Using the mesh currents defined in the figure,

$$-v_s + 20 i_{20} + \frac{1}{5\times 10^{-6}} \int_0^t \left(i_{20} - i_L \right) dt' + v_C(0) = 0 \quad \text{or}$$

$$20 i_{20} + 0.2\times 10^{6} \int_0^t i_{20} dt' - 0.2\times 10^{6} \int_0^t i_L dt' = v_s - 12 \quad [1]$$

$$-0.2\times 10^{6} \int_0^t \left(i_{20} - i_L \right) dt' - v_C(0) + 10 i_L + 8\times 10^{-3} \frac{di_L}{dt} = 0 \quad \text{or}$$

$$-0.2\times 10^{6} \int_0^t i_{20} dt' + 10 i_L + 0.2\times 10^{6} \int_0^t i_L dt' + 8\times 10^{-3} \frac{di_L}{dt} = 12 \quad [2]$$

47. $i_s(0) = 60$ mA therefore $i_2 = i_3 - i_1 = 40$ mA.

(a) By KCL, $i_s = \frac{1}{6}\int_0^t v\,dt' + i_1(0) + \frac{1}{4}\int_0^t v\,dt' + i_2(0)$

$$\frac{di_s}{dt} = \frac{1}{6}v + \frac{1}{4}v = \frac{10}{24}v$$

also,

$$\frac{di_s}{dt} = -200(0.06)e^{-200t}$$

$$\therefore \quad v(t) = -28.8e^{-200t} \text{ V}$$

(b) $i_1(t) = \frac{1}{6}\int_0^t v\,dt' + i_1(0) = -\frac{1}{6}\int_0^t 28.8e^{-200t'}dt' + 0.02 = 24e^{-200t} + 0.02 \text{ mA}$

(c) $i_2(t) = \frac{1}{4}\int_0^t v\,dt' + i_2(0) = \frac{1}{4}\int_0^t 28.8e^{-200t'}dt' + 0.040 = 36e^{-200t} + 40 \text{ mA}$

48. (a) By KVL, $v_s = v_1 + v_2$ or

$$v_s = \frac{1}{10^{-6}}\int idt + \frac{1}{4\times10^{-6}}\int idt = 100e^{-80t}$$

Thus, $i = \frac{100}{1.25\times10^{-6}}\frac{d}{dt}\left(e^{-80t}\right) = \boxed{-6.4e^{-10t}\ \text{mA}}$

(b) $v_1 = \frac{1}{10^{-6}}\int idt = -10^6\int 6.4\times10^{-3}e^{-80t}dt = \boxed{80e^{-80t}\ \text{V}}$

(c) $v_2 = \frac{1}{4\times10^{-6}}\int idt = \frac{v_1}{4} = \boxed{20e^{-80t}\ \text{V}}$

49. We first open-circuit the cosine source, leaving us with only dc sources. Thus, we may treat capacitors as open circuits, and inductors as short circuits. Hence,

$v_L' = 0$ and $v_C' = 20(0.03) + 9 = 9.6$ V

Returning to the original circuit and zeroing out the dc sources, we're left with a single source. However, the 9 V source, being replaced by a short circuit, leads us to $v_C''(t) = 0$.

$$i_L'' = -40\times10^{-3}\cos10^3 t$$

so

$$v_L'' = L\frac{di_L}{dt} = (60\times10^{-3})(-40\times10^{-3})(10^3)(-\sin10^3 t) = 2.4\sin10^3 t \text{ V}$$

Consequently, since $v_L = v_L' + v_L'' = 0 + 2.4\sin10^3 t$ V = $\boxed{2.4\sin10^3 t \text{ V}}$

and $v_C = v_C' + v_C'' = 9.6 + 0 =$ $\boxed{9.6 \text{ V}}$.

50. We define three clockwise mesh currents such that i_1 flows in the bottom left mesh, i_2 flows in the top mesh, and i_3 flows in the bottom right mesh.

By inspection, $i_1 = 0.02e^{-20t}$ A [1]

Then,

$$-v_x + 0.2v_x + 0.05\frac{d}{dt}(i_2 - i_3) + 100(i_2 - i_3) = 0$$
$$where\ v_x = 50(i_1 - i_2).$$
$$Thus,$$
$$-40i_1 + 140i_2 + 0.05\frac{di_2}{dt} - 100i_3 - 0.05\frac{di_3}{dt} = 0 \quad [2]$$

and

$$\frac{1}{10^{-6}}\int(i_3 - i_1)\,dt + 100(i_3 - i_2) + 40e^{-20t} = 0$$
$$or$$
$$-10^6\int i_1 dt - 100i_2 + 10^6\int i_3 dt + 100i_3 = -40e^{-20t} \quad [3]$$

51. (a) Assuming an ideal op amp,

$i_{C_f} = C_f \dfrac{dv_{C_f}}{dt}$. By KVL, $-v_s + v_{C_f} = 0$ so $v_{C_f} = v_s$

$$C_f \frac{dv_{C_f}}{dt} = C_f \frac{dv_s}{dt} = \frac{0 - v_{out}}{R_1}$$

Thus, $v_{out} = -R_1 C_f \dfrac{dv_s}{dt}$

52. We note that with no initial energy storage,

$$v_{out} = -\frac{1}{R_1 C_f}\int 20\times10^{-3}\sin 540t\ dt$$

$$v_{out} = -\frac{20\times10^{-3}}{\left(100\times10^{3}\right)\left(500\times10^{-6}\right)\left(540\right)}\left(-\cos 540t\right) = \boxed{7.41\cos 540t\ \text{V}}$$

53. $i_L = \frac{1}{L}\int_{0^-}^{t} v_s dt' + i_L(0^-)$. Assuming an ideal op amp,

$$\frac{1}{L}\int_{0^-}^{t} v_s dt' + i_L(0^-) = \frac{0 - v_{out}}{R_f}$$

Thus, $v_{out} = -\frac{R_f}{L}\int_{0^-}^{t} v_s dt' + R_f i_L(0^-)$

54. We note that this is a differentiator, so that

$$v_{out} = -R_f C_1 \frac{dv_s}{dt} = -(47\times 10^3)(100\times 10^{-6})\frac{dv_s}{dt} = -4.7\frac{dv_s}{dt}$$

(a) Taking the derivative,

$$\frac{dv_s}{dt} = (5\times 10^3)(20)\cos 200t = 0.1\cos 20t$$

Thus,

$$v_{out} = -4.7(0.1)\cos 20t = \boxed{-0.47\cos 20t \text{ V}}$$

(b) Taking the derivative,

$$\frac{dv_s}{dt} = -2e^{-t}$$

Thus,

$$v_{out} = -4.7\left(-2e^{-t}\right) = \boxed{9.4e^{-t} \text{ V}}$$

55. **<DESIGN>** One possible solution:

We feed a differentiator into an inverting amplifier to correct the polarity.

This yields $v_{out} = +R_f C_1 \frac{dv_s}{dt}$

We need $R_f C_1 = 1000$ so that 1 ºC/s (which corresponds to v_s changing by 1 mV per second) yields $v_{out} = 1$ V.

Choose C_1 = 1 mF, which dictates that R_f = 1 MΩ. For simplicity, choose both resistors in the inverting amplifier to also be 1 MΩ each.

56. **<DESIGN>** One possible solution:

We feed a differentiator into an inverting amplifier to correct the polarity.

This yields $v_{out} = +R_f C_1 \dfrac{dv_s}{dt}$

We need $R_f C_1 = 1$ so that 1 m/s (which corresponds to v_s changing by 1 mV per second) yields $v_{out} = 1$ mV.

Choose R_f = 1 Ω, which dictates that C_1 = 1 F. For simplicity, choose both resistors in the inverting amplifier to also be 1 Ω each.

57. **<DESIGN>** One possible solution:

We treat the proton detector as a current source, and place it in parallel with a 1 Ω resistor so that the voltage across the pair is proportional to the number of protons per second.

We connect this to the input of an integrator stage, the output of which is fed into an inverting amplifier stage to correct the polarity of the output voltage.

This yields $v_{out} = \frac{1}{R_f C_1} \int \left(I_{\det ector} \right)(1) dt$

With $\int \left(I_{\det ector} \right)(1) dt = 10^6$ hits corresponding to 1V we need

$$\frac{10^6}{R_1 C_f} = 1$$

$$or$$

$$R_1 C_f = 10^6$$

Choosing R_1 = 10 MΩ, C_f must be 100 mF. We choose the two resistors of the inverting amplifier to also be 10 MΩ each for simplicity.

58. **<DESIGN>** One possible solution:

(a) We feed a differentiator into an inverting amplifier to correct the polarity.

This yields $v_{out} = +R_f C_1 \dfrac{dv_s}{dt}$

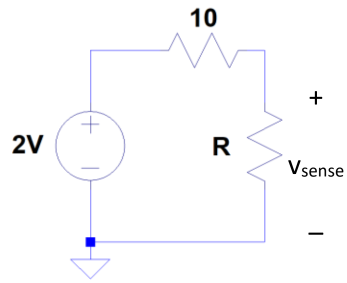
We need $R_f C_1 = 1$ so that 1 m/s^2 (which corresponds to v_s changing by 10 mV per second) yields $v_{out} = 10$ mV.

Choose $R_f = 1\ \Omega$, which dictates that $C_1 = 1$ F. For simplicity, choose both resistors in the inverting amplifier to also be 1 Ω each.

(b) Three such units would likely be needed, to detect motion in all three orthogonal directions.

59. **<DESIGN>** One possible solution:

(a) A simple voltage divider will provide us the needed voltage output, if constructed as:



Then, when R = 0, $v_{sense} = 0$ and when R = 10 Ω, $v_{sense} = (2)(1/2) = 1$ V.

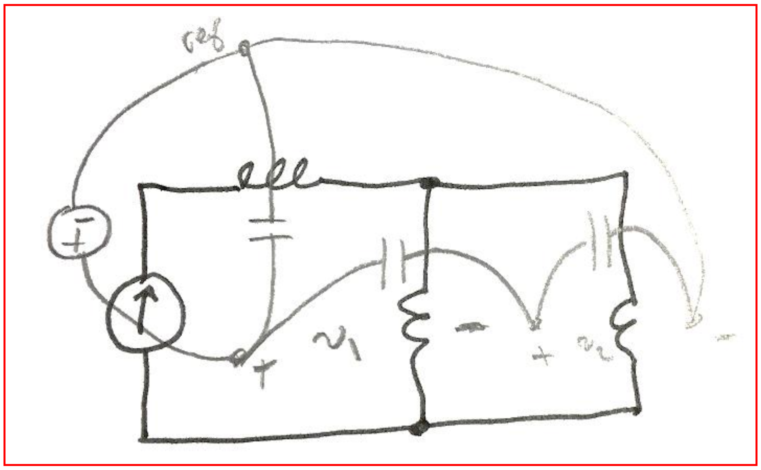
(b) We connect the positive terminal of v_{sense} to the input of a differentiator, the output of which is fed into an inverting amplifier to correct the output polarity.

This yields $v_{out} = +R_f C_1 \dfrac{dv_{sense}}{dt}$

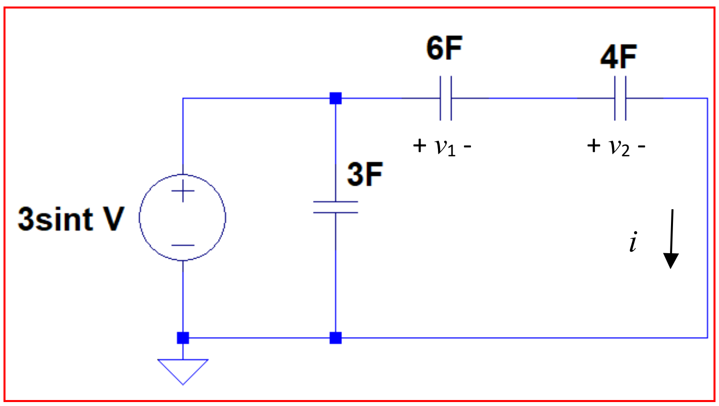
We need $R_f C_1 = 100$ so that 1 ℓ/s (which corresponds to v_{sense} changing by 10 mV per second) yields $v_{out} = 1$ V.

Choose R_f = 10 MΩ, which dictates that C_1 = 1 μF. For simplicity, choose both resistors in the inverting amplifier to also be 10 MΩ each.

60. (a) We begin by overlaying our dual directly on the original circuit:



(b) Redrawing and labelling:



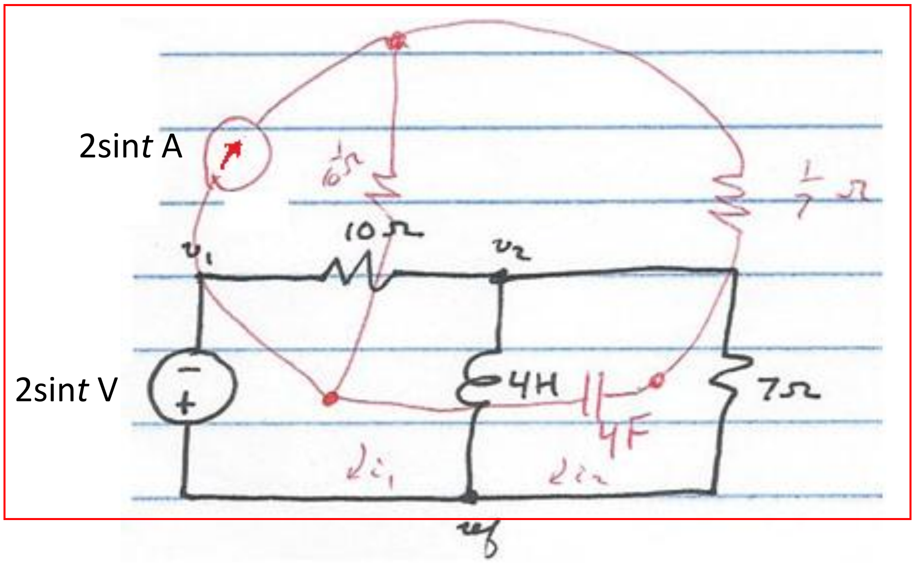
a. Original circuit, nodal analysis:

$$3\sin t = 6\frac{dv}{dt} + 4\frac{dv}{dt} = 10\frac{dv}{dt}$$

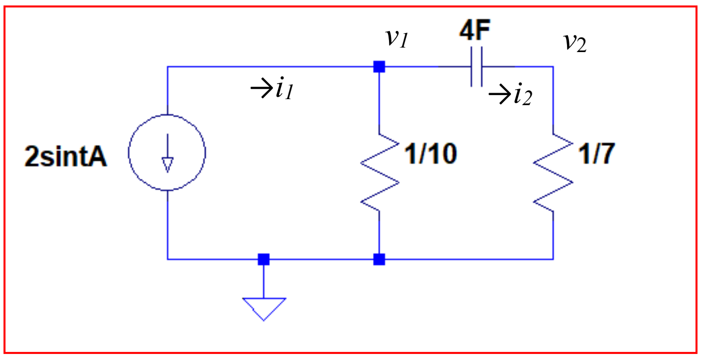
Dual circuit:

$$3\sin t = \frac{1}{6}\int_0^t i dt' + v_1(0) + \frac{1}{4}\int_0^t i dt' + v_2(0)$$

61. (a) We begin by drawing the dual circuit directly over the original circuit.



(b) We first redraw the dual circuit for clarity.



(c)

Original circuit, mesh: $2\sin t + 10 i_1 + 4\dfrac{d}{dt}(i_1 - i_2) = 0$

$$7 i_2 + 4\frac{d}{dt}(i_2 - i_1) = 0$$

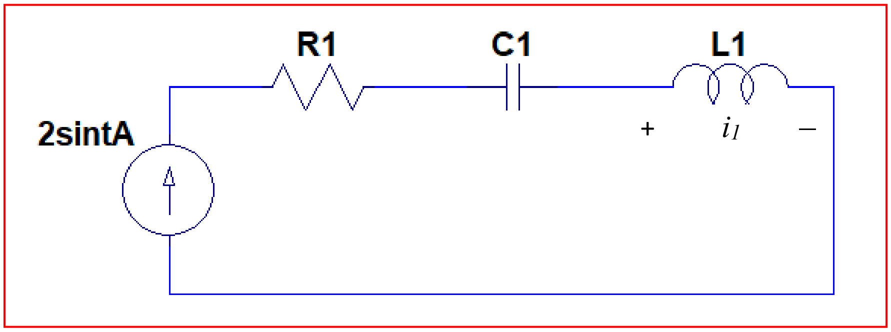
Nodal: v_1 = - 2sin$t$ V

$$0 = \frac{v_2 - v_1}{10} + \frac{v_2}{7} + \frac{1}{4}\int_{0^-}^{t} v_2\,dt' + i_1(0^-) - i_2(0^-)$$

New, mesh: i_1 = -2sin$t$ A

$$0 = \left(\frac{1}{7} + \frac{1}{10}\right) i_2 - \frac{1}{10} i_1 + \frac{1}{4}\int_{0^-}^{t} i_2\,dt' + v_1(0^-) - v_2(0^-)$$

62. (a) With four elements in parallel, we expect our dual to contain four (dual) elements in series. Thus,



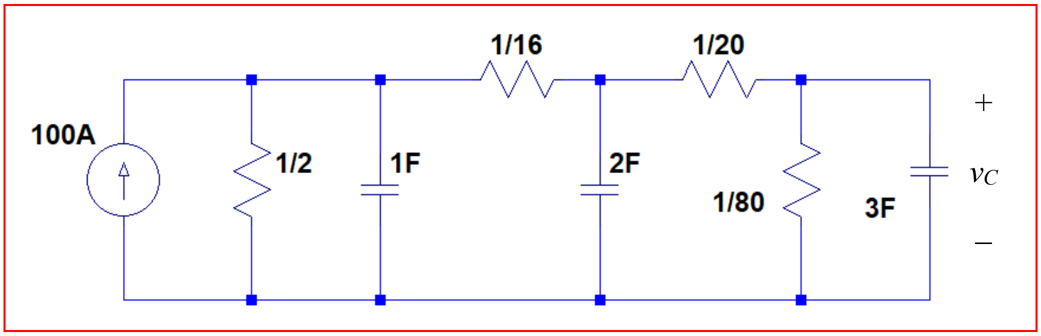
b. Original circuit: define v_1 as top node, and bottom node as reference. Then,

$$v_1 = 2\sin t \text{ V by inspection}$$

Dual circuit:

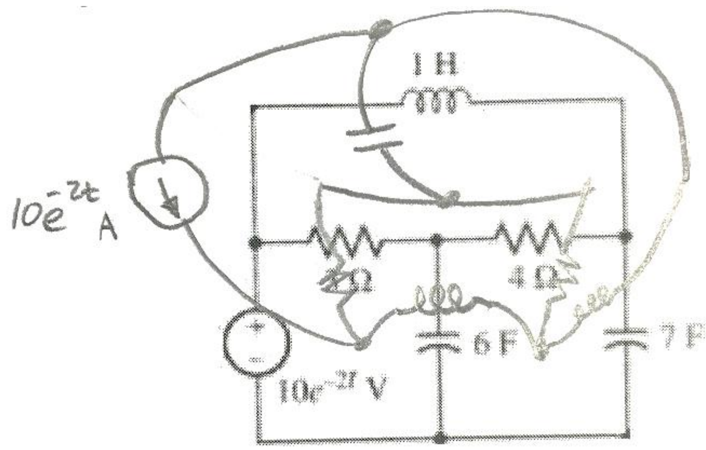
$$i_L = 2\sin t \text{ A by inspection}$$

63. (a) We construct the dual by noting that elements in series will be represented by their duals in parallel, and vice-versa. Thus,

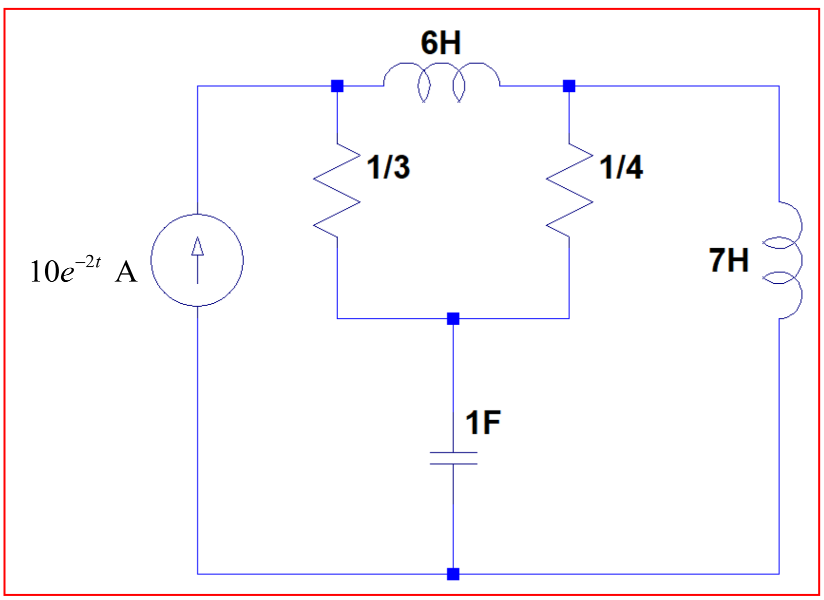


(b) The current dual is a voltage, as labelled.

64. We begin by overlaying our dual circuit directly on top of the original circuit, using it as a guide.



Finally, we redraw and label each dual element:

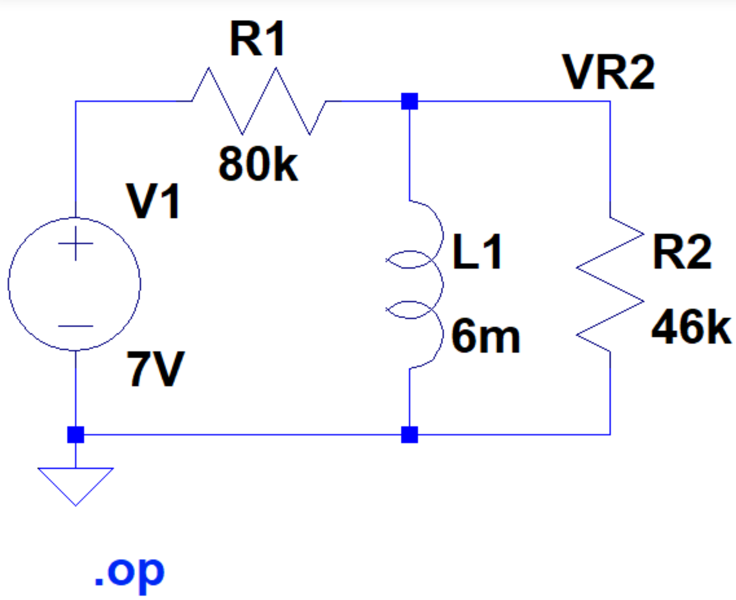


65. (a) DC. Thus, $i_L = 7/80\times10^3$ = 87.5 μA

(b) P_{diss} = 0

(c) We see from the dc simulation that the inductor current is 87.5 mA as computed, and there is essentially zero current flowing through the 46 kΩ resistor, hence it dissipates 0 W as expected.

```
        --- Operating Point ---

V(n001):        7              voltage
V(vr2):         8.75e-008      voltage
I(L1):          8.75e-005      device_current
I(R2):          1.90217e-012   device_current
I(R1):          -8.75e-005     device_current
I(V1):          -8.75e-005     device_current
```



66. (a) At DC, the capacitor is an open circuit. Hence, the current through each resistor is given by $i = \frac{7}{80+46}$ mA $= 55.6\ \mu$A . Thus

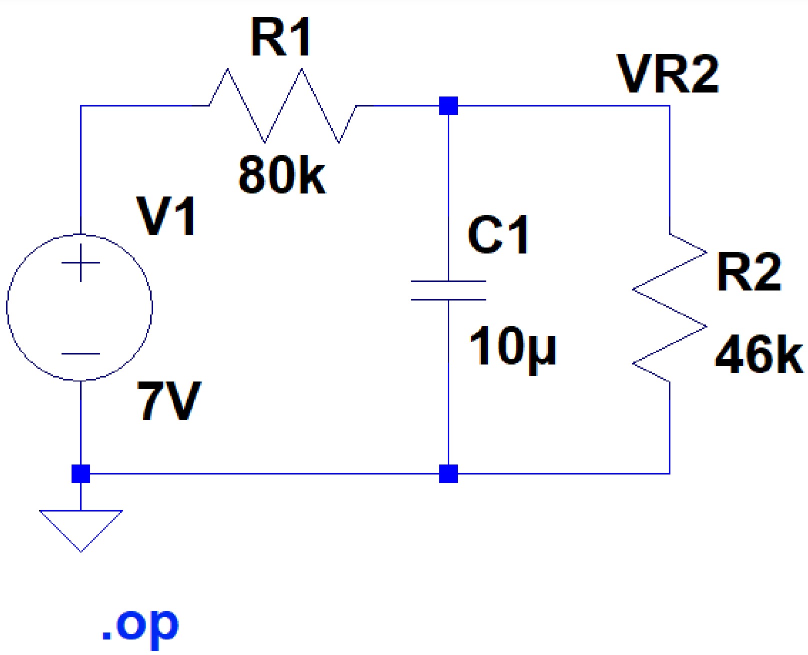
$P_{80k} = (55.6\times10^{-6})^2(80\times10^3) =$ **247 mW**

$P4_{6k} = (55.6\times10^{-6})^2(46\times10^3) =$ **142 mW**

(b) $v_C = 7\frac{46}{80+46} = 2.56$ V. Thus, $w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(10\times10^{-6})(2.56)^2 =$ **32.8 μJ**

(c) Our simulation confirms both the resistor currents and capacitor voltage:

```
       --- Operating Point ---

V(n001):       7              voltage
V(vr2):        2.55556        voltage
I(C1):         2.55556e-017   device_current
I(R2):         5.55556e-005   device_current
I(R1):         -5.55556e-005  device_current
I(V1):         -5.55556e-005  device_current
```



67. (a) At DC, the inductor is a short circuit, and the capacitor is an open circuit.

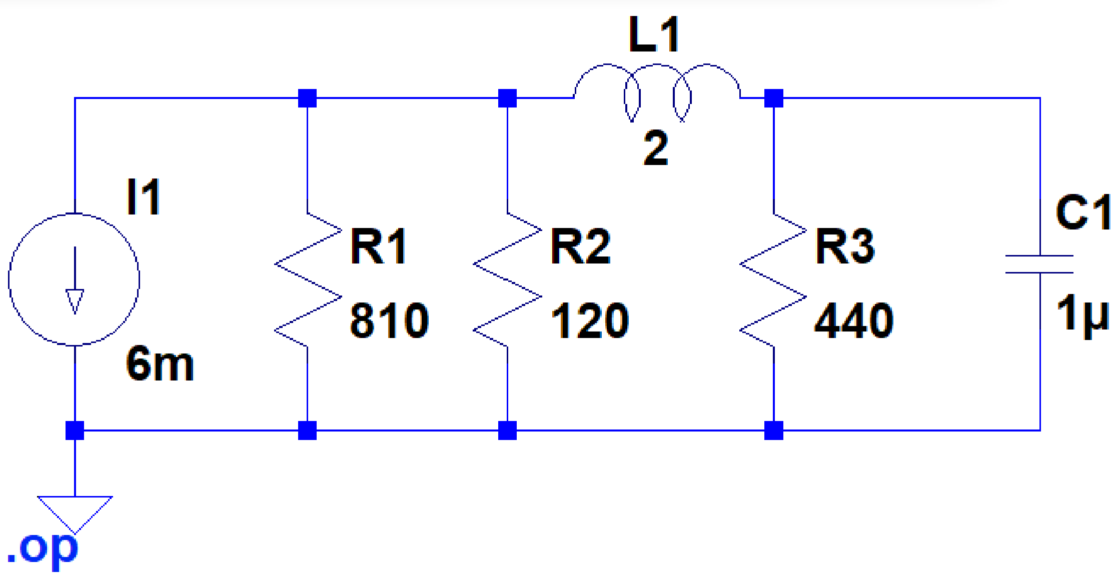
By current division, $i_L = -6\times10^{-3}\dfrac{\frac{1}{440}}{\frac{1}{810}+\frac{1}{120}+\frac{1}{440}} = \boxed{-11.52 \text{ mA}}$.

Thus, $v_x = (440(i_L) = -5.069$ V

(b) $w_L = \frac{1}{2}Li^2 = \boxed{132.7\ \mu J}$. $w_C = \frac{1}{2}Cv_x^{\ 2} = \boxed{12.85\ \mu J}$

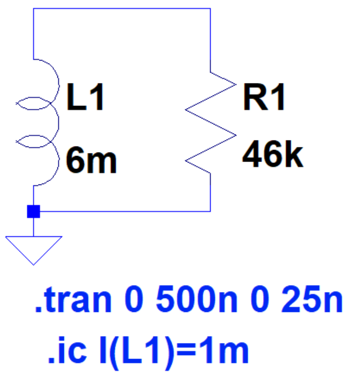
(c) Our simulation confirms both the inductor current and capacitor voltage (the sign of the inductor current in the simulation is an artifact of how the component was rotated before placing):

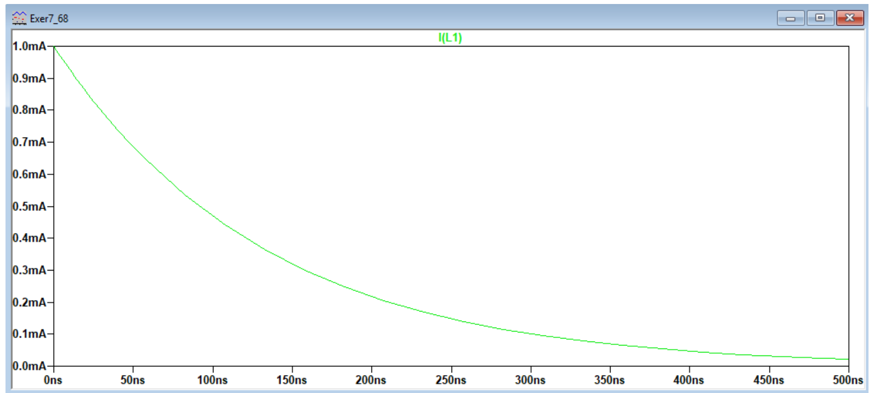
```
        --- Operating Point ---

V(n001):        -0.50673        voltage
V(n002):        -0.506729       voltage
I(C1):          -5.06729e-019   device_current
I(L1):          -0.00115166     device_current
I(I1):          0.006           device_current
I(R3):          -0.00115166     device_current
I(R2):          -0.00422275     device_current
I(R1):          -0.000625593    device_current
```



68. (a) $w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(6\times10^{-3})(1\times10^{-3})^2 = \boxed{3 \text{ nJ}}$

(b) We set up the transient simulation as:





Using the cursor, we complete the table below:

t (ns)	i_L(μA)
0	999.99
130	372.96
260	136.81
500	21.40

(c) $w_L = \frac{1}{2}Li_L^2$ so

using the table above,

at t = 130 ns, $w_L = \frac{1}{2}(6\times10^{-3})(372.96\times10^{-6})^2 = \boxed{3.83\ \mu J}$

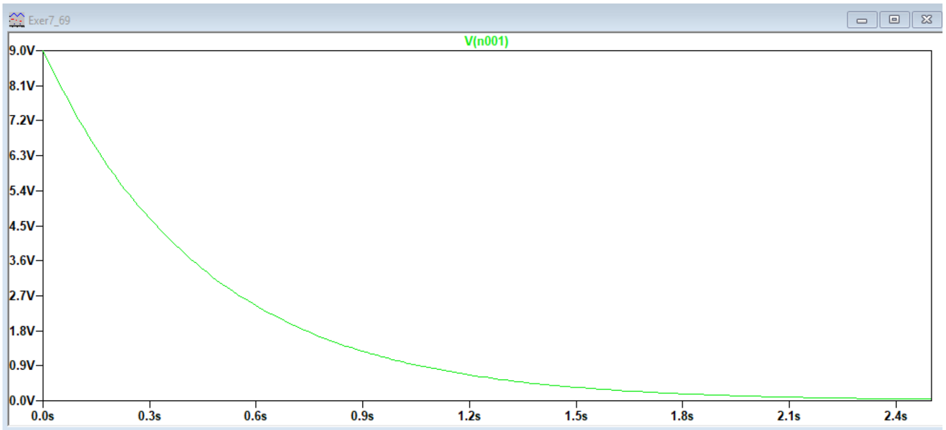
and

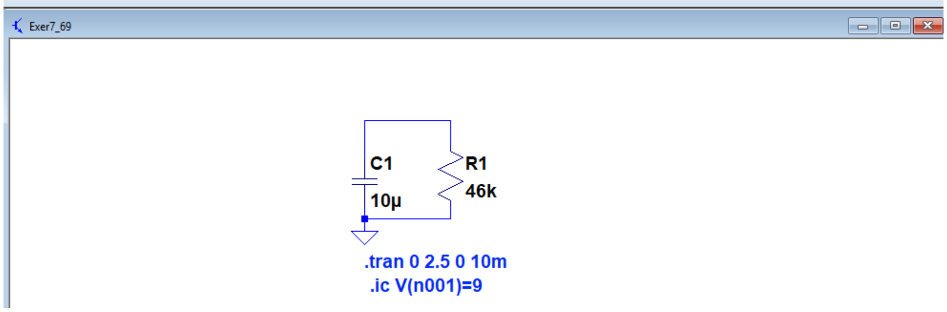
at t = 500 ns, $w_L = \frac{1}{2}(6\times10^{-3})(21.4\times10^{-6})^2 = \boxed{1.38\ pJ}$

69. (a) $w = (\frac{1}{2})(10\times10^{-6})(81) = 405\ \mu\text{J}$

(b) no - the resistor will slowly dissipate the energy stored in the capacitor

(c) We perform the following simulation:





Using the cursor, we complete the table below:

t (ms)	v_C (V)
0	9
460	3.31
920	1.22
2300	60.4×10^{-3}

(d) Fraction of energy at selected times:

$w_C = \frac{1}{2}Cv_C^2$ so using the table above,

at t = 460 ms, $w_C = \frac{1}{2}(10\times10^{-6})(3.31)^2 = 110\ \mu\text{J}$ which is 100(110/(405) = 27%

and

at t = 2.3 s, $w_C = \frac{1}{2}(10\times10^{-6})(60.4\times10^{-3})^2 = 18.2\ n\text{J}$ which is 100(0.0182)/405 =

0.004%

70. (a) Define three clockwise mesh currents: i_1 in the left bottom mesh, i_3 in the top mesh, and i_2 in the bottom right mesh (noting the open circuited capacitor means we can treat the 2 Ω resistor and the current source as having the same mesh current).

$$-4 + 1(i_1 - i_3) + 2(i_1 - i_2) = 0 \quad [1]$$

$$i_2 = -0.002 \quad \text{(by inspection)} \quad [2]$$

$$-5v_x + 1(i_3 - i_1) = 0 \quad [3]$$

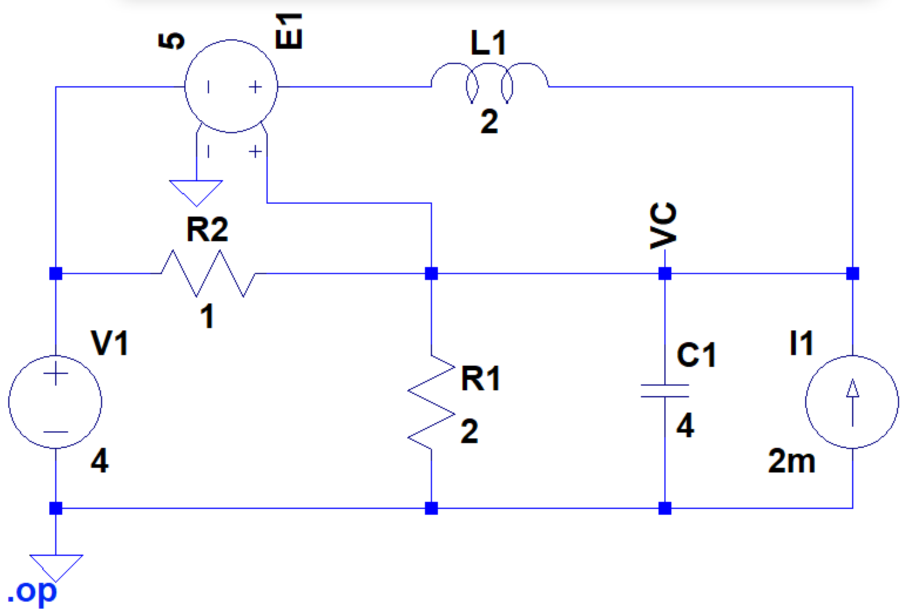
$$v_x = 2(i_1 - i_2) \quad [4]$$

Solving, v_x = -1.00 V and $i_L = i_3$ = 5.502 A.

Then, $w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(4)(1.00)^2 = \boxed{2\text{ J}}$ and $w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(5.50)^2 = \boxed{30.3\text{ J}}$

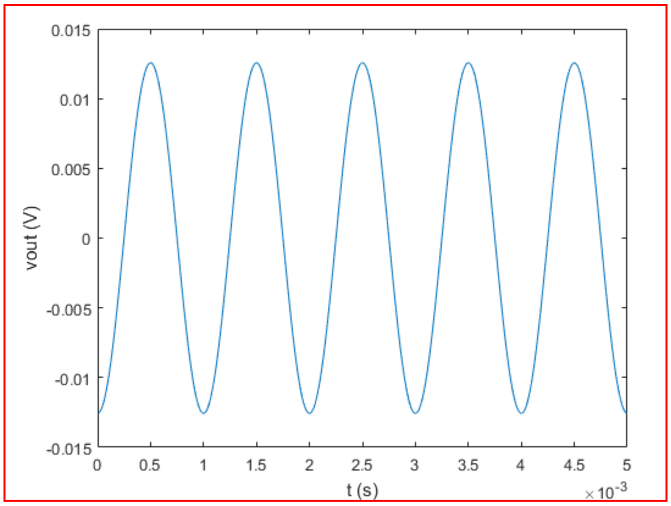
(b) We see that our simulation agrees well with the hand calculations (5.50 A, -1.00 V).

```
--- Operating Point ---

V(n002):     -1.00688       voltage
V(vc):       -1.00138       voltage
V(n001):     4              voltage
I(C1):       -4.0055e-012   device_current
I(L1):       -5.50406       device_current
I(I1):       0.002          device_current
I(R2):       -5.00138       device_current
I(R1):       -0.500688      device_current
```

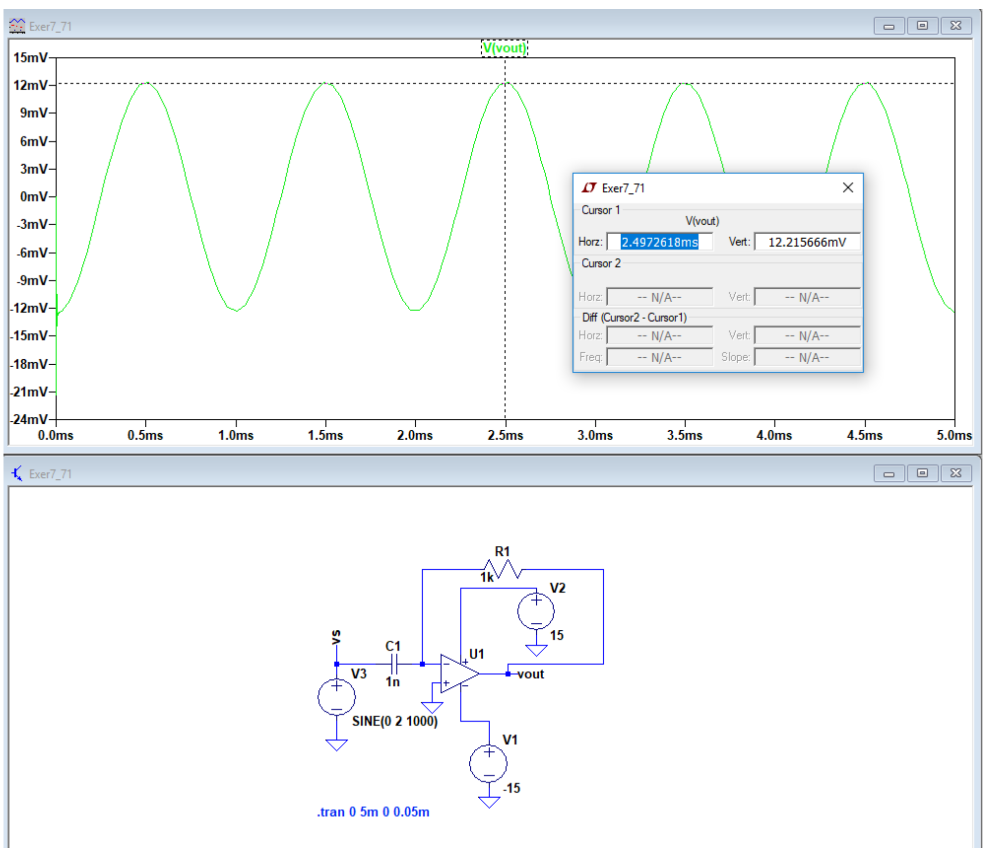


71. (a) $v_s = 2\sin(2\pi\times10^3 t)$ V so

$$v_{out} = -R_f C_1 \frac{dv_s}{dt} = -(10^3)(10^{-9})(2\pi\times10^3)\cos(2\pi\times10^3 t) \text{ V} = -12.6\cos(2\pi\times10^3 t) \text{ mV}$$

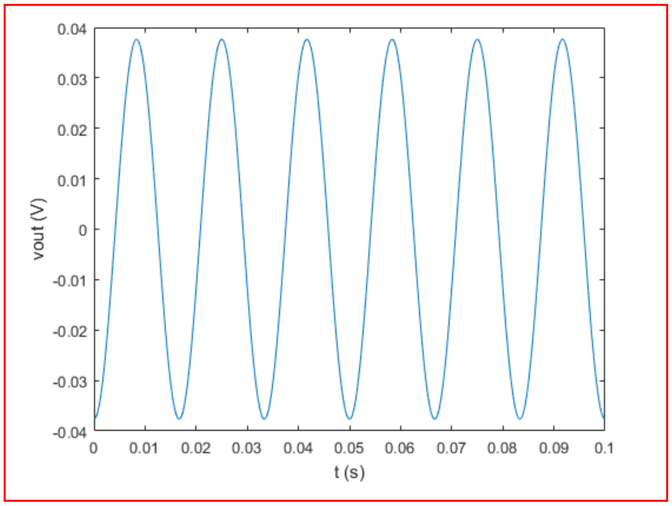


(b) Within a reasonable amount of rounding error, we see that our simulation agrees well with our hand calculation in terms of amplitude.

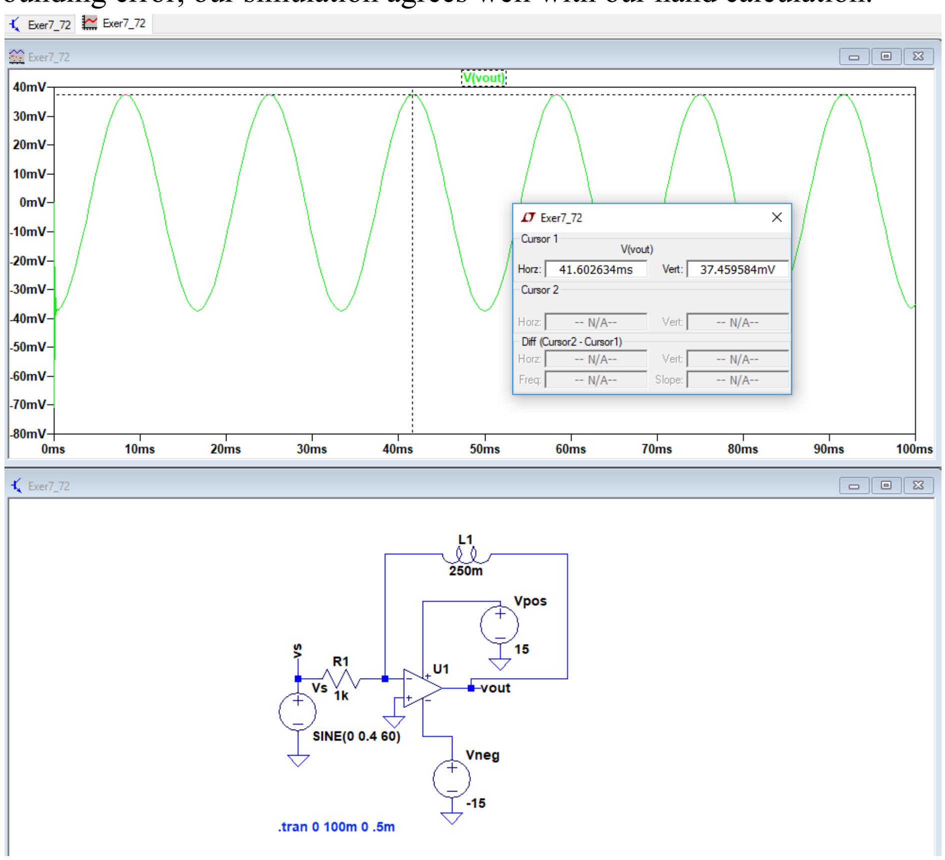


72. (a) $v_s = 0.4\sin(120\pi t)$ V so

$$v_{out} = -\frac{L_f}{R_1}\frac{dv_s}{dt} = -\frac{250\times10^{-3}}{1000}(120\pi)(0.4)\cos(120\pi t)\ \text{V} \;=\; -37.7\cos(120\pi t)\ \text{mV}$$



(b) Within rounding error, our simulation agrees well with our hand calculation.

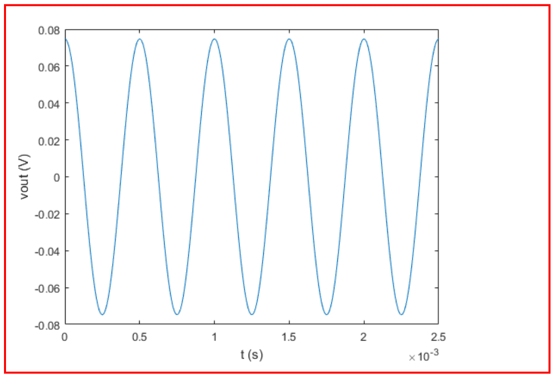


73. The circuit in Fig. 7.72 is an integrator: If we assume an ideal op amp with no initial energy stored in the inductor, then

$$i_L = \frac{1}{L}\int_{0^-}^{t} v_s dt' \quad \text{and equating currents leads to} \quad \frac{1}{L}\int v_s dt = \frac{0 - v_{out}}{R_f}$$

and, $v_{out} = -\frac{R_f}{L}\int v_s dt$. With $R_f = 47\ \Omega$, $L_1 = 0.1$ H and $v_s = 2\sin(4\pi\times10^3 t)$ V we find

$$v_{out} = -\frac{47}{0.1}\int 2\sin\left(4\pi\times10^3 t\right)dt = \frac{47(2)}{(0.1)(4\pi\times10^3)}\cos\left(4\pi\times10^3\right) = 74.8\cos\left(4\pi\times10^3\right)\ \text{mV}$$



(b) Our simulation agrees well in terms of magnitude (74.5 mV = 148.9/2) but contains an unexpected dc offset.