

1. (a)  $f_{\text{linear}} = 1 + x$

(b) Define %error as  $100 \times [e^x - (1+x)]/e^x$ 

$x$	$f_{\text{linear}}$	$e^x$	%error
$5 \times 10^{-6}$	1.000005	1.000005000	-
$5 \times 10^{-4}$	1.0005	1.0005001	$10^{-5}\%$
$5 \times 10^{-2}$	1.05	1.05127	0.1%
0.5	1.5	1.649	9.0%
5	6	148.4	96%

(c) Somewhat subjectively, we note that the relative error is  $\sim 0.1\%$  or better for  $x \leq 0.05$  so use this as our estimate of what constitutes “reasonable.”

2.  $y(t) = 4 \sin 2t \approx 4(2t) = 8t$

(a) Define %error  $\square 100 \times \left| \frac{8t - 4 \sin 2t}{4 \sin 2t} \right|$

$t$	$8t$	$4 \sin 2t$	%error
$10^{-6}$	$8 \times 10^{-6}$	$8.000 \times 10^{-6}$	0% (to 4 digits)
$10^{-4}$	$8 \times 10^{-4}$	$8.000 \times 10^{-4}$	0% (to 4 digits)
$10^{-2}$	$8 \times 10^{-2}$	0.07999	0.01%
$10^{-1}$	$8 \times 10^{-1}$	0.7947	0.7%
1.0	8.0	3.637	120%

(b) This linear approximation holds well ( $< 1\%$  relative error) even up to  $t = 0.1$ . Above that value and the errors are appreciable.

$$3. \quad i_8|_{6 \text{ A only}} = 6 \frac{3}{3+8} = \frac{18}{11} \text{ A}. \quad i_8|_{2 \text{ V only}} = 2 \frac{1}{3+8} = \frac{2}{11} \text{ A}$$

4. (a) We replace the voltage source with a short circuit and designate the downward current through the  $4\ \Omega$  resistor as  $i'$ .

$$\text{Then, } i' = (10)(9)/(13) = 6.923\ \text{A}$$

Next, we replace the current source in the original circuit with an open circuit and designate the downward current through the  $4\ \Omega$  resistor as  $i''$ .

$$\text{Then, } i'' = 1/13 = 0.07692\ \text{A}$$

$$\text{Adding, } i = i' + i'' = 7.000\ \text{A}$$

- (b) The 1 V source contributes  $(100)(0.07692)/7.000 = 1.1\%$  of the total current.

(c)  $I_x(9)/13 = 0.07692$ . Thus,  $I_x = 111.1\ \text{mA}$

5. (a) Open circuit the current source, and note that  $2\ \Omega \parallel 2\ \Omega = 1\ \Omega$ .

$$i'_x = \frac{(2)\left(\frac{1}{2+1}\right)}{2} = \frac{1}{3}\text{ A}$$

Now, short circuit the voltage source instead.

$$i''_x = (1)\frac{\left(\frac{1}{2}\right)}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{1}{3}\text{ A}$$

$$\text{Combining, } i_x = i'_x + i''_x = \frac{2}{3}\text{ A}$$

- (b) Each source contributes equally, hence 50%.

- (c) To obtain  $i_x = \frac{4}{3}\text{ A}$ ,  $i''_x$  must provide 1 A, so increase the current source to 3 times its current value, or 3 A.

6. (a) Open circuiting the 4 A source leaves  $5 + 5 + 2 = 12 \Omega$  in parallel with the  $1 \Omega$  resistor. Thus,  $v_1|_{7\text{ A}} = (7)(1||12) = (7)(0.9231) = 6.462 \text{ V}$

Open circuiting the 7 A source leaves  $1 + 5 = 6 \Omega$  in parallel with  $5 + 2 = 7 \Omega$ . Assisted by current division,

$$v_1|_{4\text{ A}} = (1) \left[ -4 \frac{7}{6+7} \right] = -2.154 \text{ V}$$

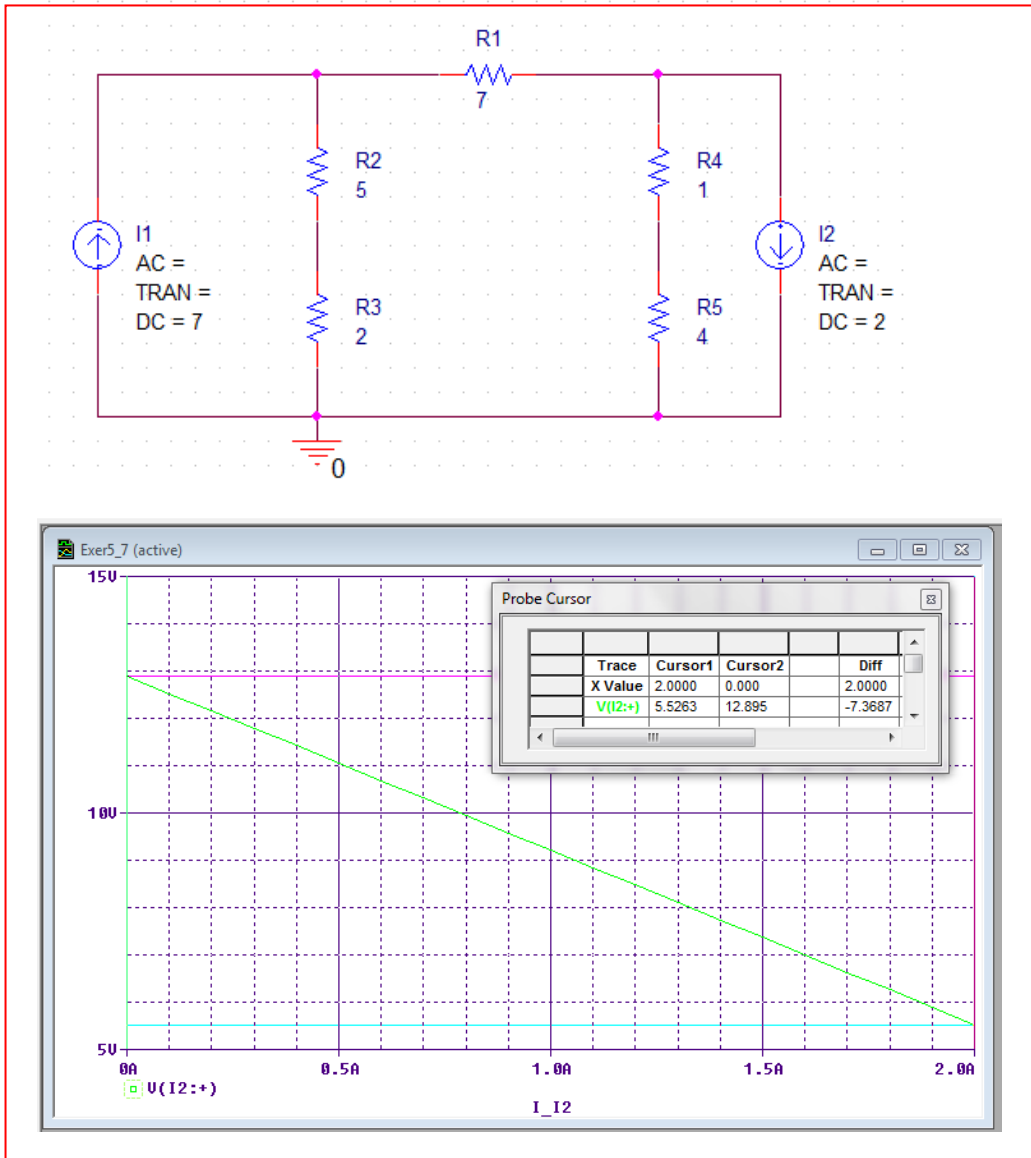
Thus,  $v_1 = 6.462 - 2.154 = 4.308 \text{ V}$

$$(b) P_{1\Omega} = \frac{(v_1)^2}{1} = (4.308)^2 = 18.56 \text{ W}$$

7. (a)  $v_2|_{7A} = (5) \left[ 7 \frac{7}{7+7+5} \right] = 12.89 \text{ V}$

$$v_2|_{2A} = (5) \left[ -2 \frac{7+5+2}{14+5} \right] = -7.368 \text{ V}$$

(b) We see from the simulation output that the 7 A source alone contributes 12.89 V. The output with both sources on is 5.526 V, which agrees within rounding error to our hand calculations (5.522 V).



8. (a)  $4\text{ V} \rightarrow 8\text{ V}; 10\text{ V} \rightarrow 20\text{ V}$
- (b)  $4\text{ V} \rightarrow -4\text{ V}; 10\text{ V} \rightarrow -10\text{ V}$

$$9. \quad v_x = 12 \frac{3 \parallel 2}{(3 \parallel 2) + 1} + (-15) \frac{1 \parallel 3}{(1 \parallel 3) + 2} = \boxed{2.455 \text{ V}}$$

$$v_x' = 6 \frac{3 \parallel 2}{(3 \parallel 2) + 1} + (-10) \frac{1 \parallel 3}{(1 \parallel 3) + 2} = \boxed{0.5455 \text{ V}}$$

$$v_x'' = 6 \frac{3 \parallel 2}{(3 \parallel 2) + 1} + (-5) \frac{1 \parallel 3}{(1 \parallel 3) + 2} = \boxed{1.909 \text{ V}}$$

$$v_x' + v_x'' = 2.455 \text{ V (as anticipated)}$$

10. (a) With the right-hand voltage source short-circuited and the current source open-circuited, we have  $2 \parallel 5 = 10/7 \Omega$

$$\text{By voltage division, } v_x|_{\text{lefthand 4 V}} = (4) \frac{1}{3+1+10/7} = 0.7368 \text{ V}$$

With the other voltage source short-circuited and the current source open-circuited, we have  $(3+1) \parallel 5 = 2.222 \Omega$ .

$$v_{5\Omega} = 4 \frac{2.222}{2.222+2} = 2.105 \text{ V. Then, } v_x|_{\text{righthand 4 V}} = -2.105 \frac{1}{4} = -0.5263 \text{ V}$$

Finally, with both voltage sources short-circuited, we find that

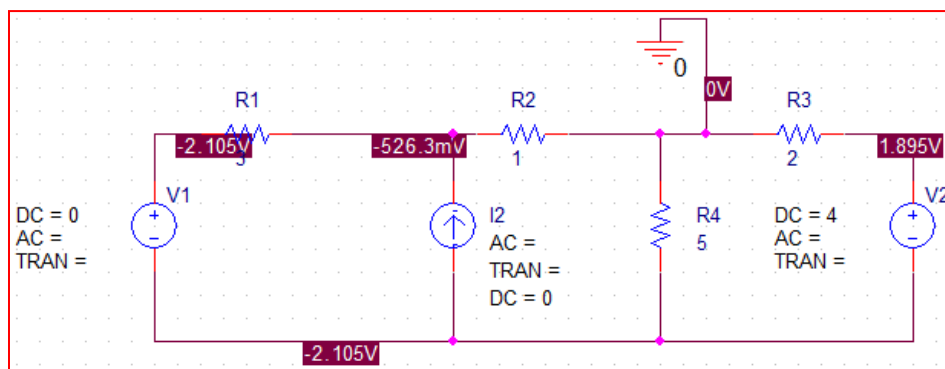
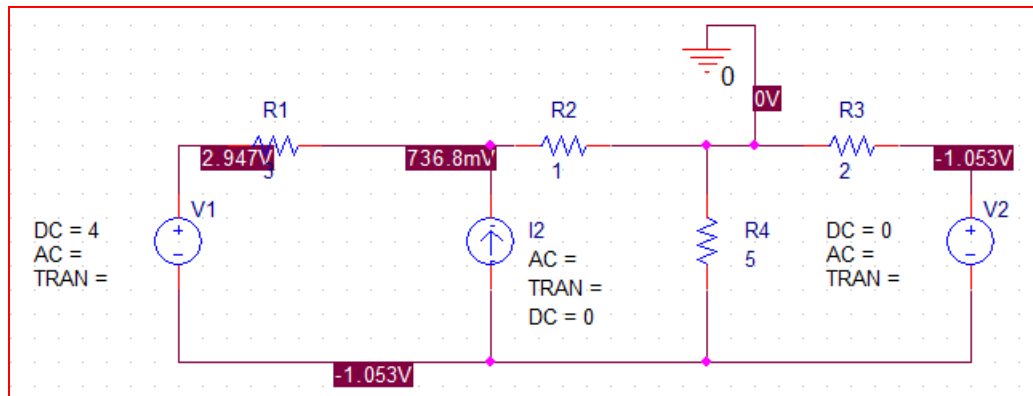
$$v_x|_{2\text{A}} = (1) \left[ 2 \frac{3}{3+1+10/7} \right] = 1.105 \text{ V}$$

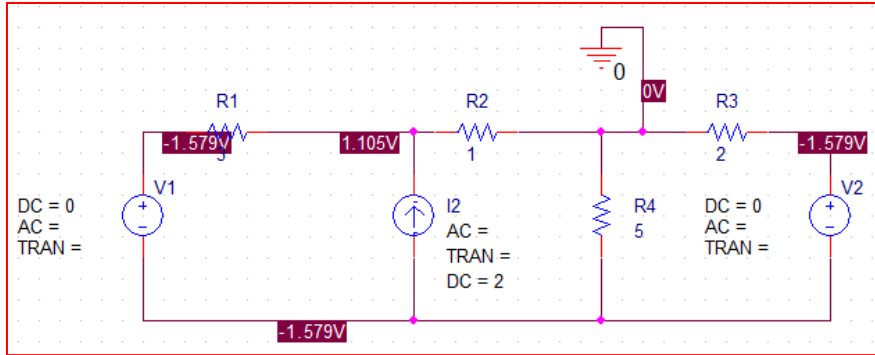
Adding these three terms together,  $v_x = 1.316 \text{ V}$

$$(b) (0.9)(1.316) = 0.7368 + 1.105k - 0.5263$$

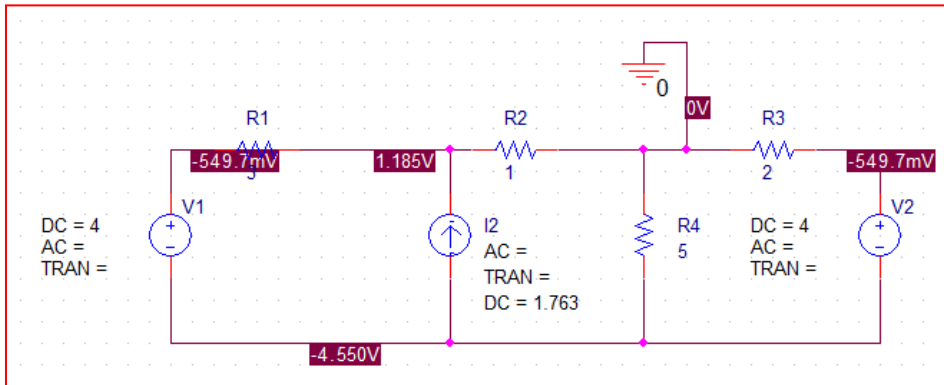
Solving,  $k = 0.8814$ . Hence, we should reduce the 2 A source to  $2k = 1.763 \text{ A}$

(c) Our three separate simulations:





Our reduced voltage alternative:



11. We select the bottom node as the reference, then identify  $v_1$  with the lefthand terminal of the dependent source and  $v_2$  with the righthand terminal.

Via superposition, we first consider the contribution of the 1 V source:

$$\frac{v_1' - 1}{5000} + \frac{v_1'}{7000} + \frac{v_2'}{2000} = 0 \quad \text{and}$$

$$\left(1 + \frac{0.2}{7000}\right)v_1' - v_2' = 0$$

Solving,  $v_1' = 0.237 \text{ V}$

Next, we consider the contribution of the 2 A source:

$$\frac{v_1''}{5000} + \frac{v_1''}{7000} + \frac{v_2''}{2000} = -2 \quad \text{and}$$

$$\left(1 + \frac{0.2}{7000}\right)v_1'' - v_2'' = 0$$

Solving,  $v_1'' = -2373 \text{ V}$ . Adding our two components,  $v_1 = -2373 \text{ V}$ .

Thus,  $i_x = v_1/7000 = -339 \text{ mA}$

12. (a) Begin by short-circuiting the voltage source.

$$\text{KCL yields } 2 = i'_x + 0.2i'_x \quad \text{or } i'_x = \frac{2}{1.2} = 1.667 \text{ A}$$

Now, open-circuit the independent current source instead.

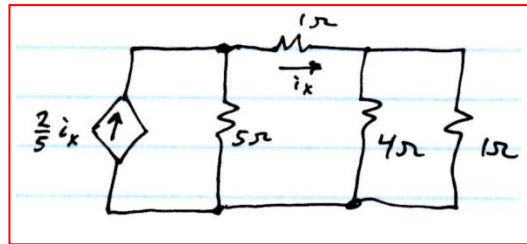
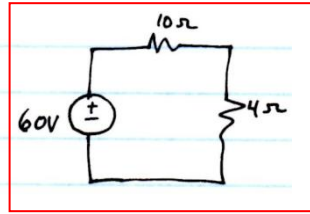
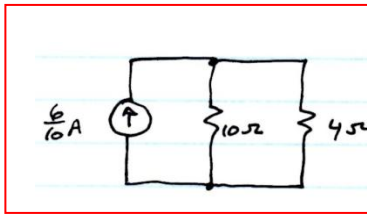
$$i''_x = -0.2i''_x \quad \text{therefore } i''_x = 0$$

$$\text{Hence, } i_x = i'_x + i''_x = 1.667 \text{ A}$$

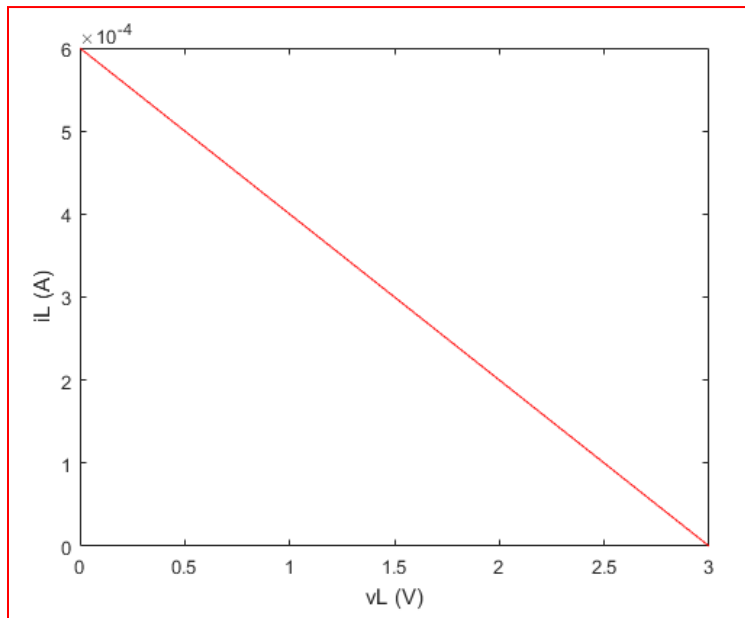
(b)  $P_{1\Omega(\text{left})} = (1)(0.2i_x)^2 = 111 \text{ mW}$

$$P_{1\Omega(\text{right})} = \frac{(2)^2}{1} = 4 \text{ W}$$

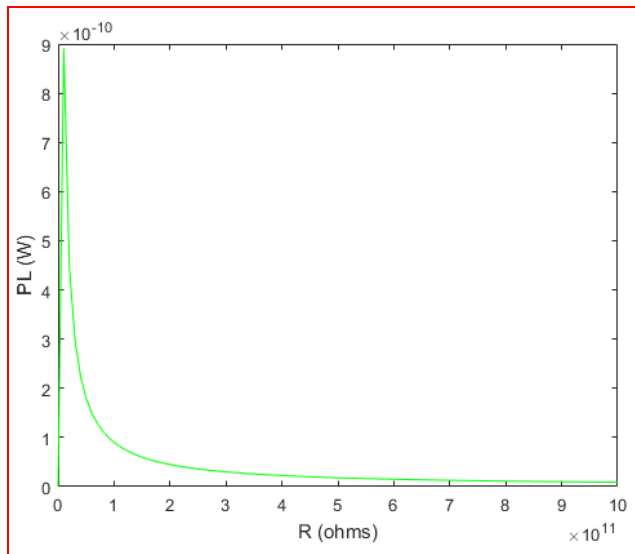
13.



14. (a)  $v_L = 3 \frac{R}{R+5000}$ ;  $i_L = \frac{3}{5000+R}$



(b)  $i_L = \frac{3}{5000+R}$  so  $P_R = i_L^2 R = \left[ \frac{9}{(5000+R)^2} \right] R$



(c)  $i_L = \left( \frac{3}{5000} \right) \left( \frac{5000}{5000+R} \right) = \frac{3}{5000+R}$  so  $P_R = i_L^2 R = \left[ \frac{9}{(5000+R)^2} \right] R$  as before. Thus,

the plot will be unchanged. We note a clear maximum value at  $R = 5 \text{ k}\Omega$ .

15. We cannot involve the  $5\ \Omega$  resistor in any transforms as we are interested in its current.

Hence, combine the  $7$  and  $4\ \Omega$  to obtain  $11\ \Omega$ ; Transform to  $9/11$  A current source in parallel with  $11\ \Omega$ .

$3 + 9/11 = 42/11$  in parallel with  $1\ \Omega$ .

No further simplification is advisable although  $5 \parallel 11 = 3.44\ \Omega$ . Hence,

$$V_{5\Omega} = (42/11)(3.44) = 13.13\ \text{V so } I = 13.13/5 = 2.63\ \text{A}$$

16. For the circuit depicted in Fig. 5.22a,  $i_{7\Omega} = (5 - 3)/7 = 285.7 \text{ mA}$ .

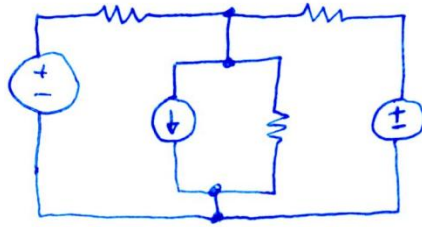
$$\text{Thus, } P_{7\Omega} = 7 \cdot (i_{7\Omega})^2 = 571 \text{ mW.}$$

For the circuit depicted in Fig. 5.22c,  $i_{7\Omega} = (5 - 3)/7 = 285.7 \text{ mA}$ .

$$\text{Thus, } P_{7\Omega} = 7 \cdot (i_{7\Omega})^2 = 571 \text{ mW.}$$

The power dissipated by this resistor is unchanged since it is proportional to  $(i_{7\Omega})^2$  which is unchanged.

17. (a) The transform available to us is clearer if we first redraw the circuit:



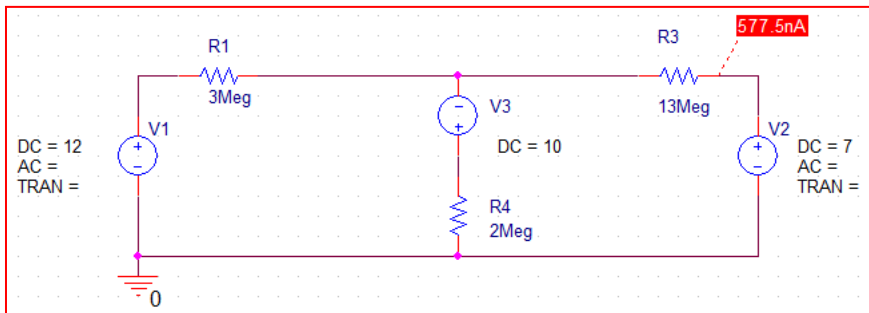
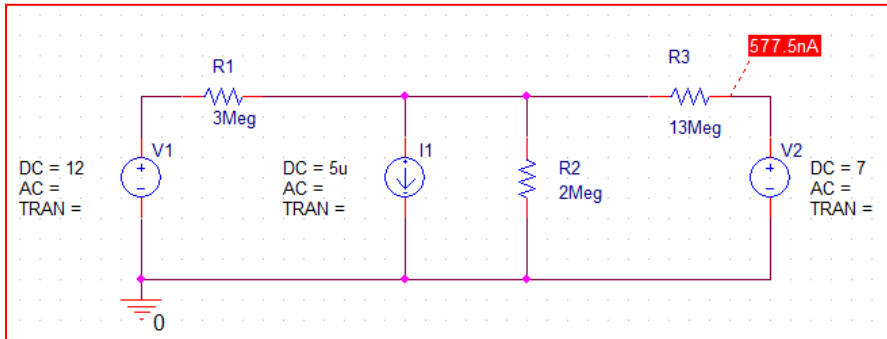
We can replace the current source / resistor parallel combination with a 10 V voltage source (“-“ terminal at the top node) in series with a 2 MΩ resistor. The circuit is easily analyzed with mesh analysis:

$$-22 + 5 \times 10^6 i_1 - 2 \times 10^6 i = 0 \quad [1]$$

$$17 + 15 \times 10^6 i - 2 \times 10^6 i_1 = 0 \quad [2]$$

Solving,  $i = -577.5 \text{ nA}$

- (b)



18. (a) Perform the following steps in order:

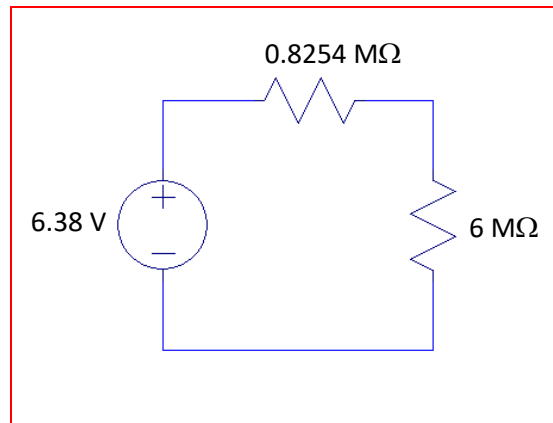
Combine the 27  $\mu\text{A}$  and 750  $\text{k}\Omega$  to obtain 20.25 V in series with 750  $\text{k}\Omega$  in series with 3.5  $\text{M}\Omega$ .

Convert this series combination to a 4.25  $\text{M}\Omega$  resistor in parallel with a 4.765  $\mu\text{A}$  source, arrow up.

Convert the 15 V/ 1.2  $\text{M}\Omega$  series combination into a 12.5  $\mu\text{A}$  source (arrow down) in parallel with 1.2  $\text{M}\Omega$ . This appears in parallel with the current source from above as well as the 7  $\text{M}\Omega$  and 6  $\text{M}\Omega$ .

Combine:  $4.25 \text{ M}\Omega \parallel 1.2 \text{ M}\Omega \parallel 7 \text{ M}\Omega = 0.8254 \text{ M}\Omega$ . This, along with the  $-12.5 \mu\text{A} + 4.765 \mu\text{A}$  yield a  $-7.735 \mu\text{A}$  source (arrow up) in parallel with 825.4  $\text{k}\Omega$  in parallel with 6  $\text{M}\Omega$ .

Convert the current source and 825.4  $\text{k}\Omega$  resistor into a 6.38 V source ('+' reference up) in series with 825.4  $\text{k}\Omega$  and 6  $\text{M}\Omega$ .



$$(b) P_{6\text{M}\Omega} = \left[ \frac{6.38}{6 \times 10^6 + 825.4 \times 10^3} \right]^2 (6 \times 10^6) = 5.24 \mu\text{W}$$

19. (a) We combine the  $1\ \Omega$  and  $3\ \Omega$  resistors to obtain  $0.75\ \Omega$ . The  $2\ \text{A}$  and  $5\ \text{A}$  current sources can be combined to yield a  $3\ \text{A}$  source.

These two elements can be source-transformed to a  $(9/4)\ \text{V}$  voltage source (“+” sign up) in series with a  $0.75\ \Omega$  resistor in series with the  $7\ \text{V}$  source and the far-left  $3\ \Omega$  resistor.

(b) In the original circuit, we define the top node of the current sources as  $v_1$  and the bottom node is our reference node.

Then nodal analysis yields  $(v_1 + 7)/3 + v_1/1 + v_1/3 = 5 - 2$

Solving,  $v_1 = 2/5\ \text{V}$  and so the clockwise current flowing through the  $7\ \text{V}$  source is

$i = (-7 - v_1)/3 = -37/15$ . Hence,  $P_{7\text{V}} = 17.27\ \text{W}$

Analyzing our transformed circuit, the clockwise current flowing through the  $7\ \text{V}$  source is  $(-7 - 9/4)/3.75 = -37/15\ \text{A}$ . (as before)

Again,  $P_{7\text{V}} = 17.27\ \text{W}$ .

20. (a) We start at the left, switching between voltage and current sources as we progressively combine resistors.

$$12/47 = 0.2553 \text{ A in parallel with } 47 \Omega \text{ and } 22 \Omega$$

$$47 \parallel 22 = 15 \Omega$$

Back to voltage source:  $(0.2553)(15) = 3.83 \text{ V}$  in series with  $15 \Omega$ .

Combine with  $10 \Omega$  to obtain  $25 \Omega$ . Back to current source:  $3.83/25 = 0.153 \text{ A}$  in parallel with  $25 \Omega$  and  $7 \Omega$ . Note that  $25 \parallel 7 = 5.47 \Omega$

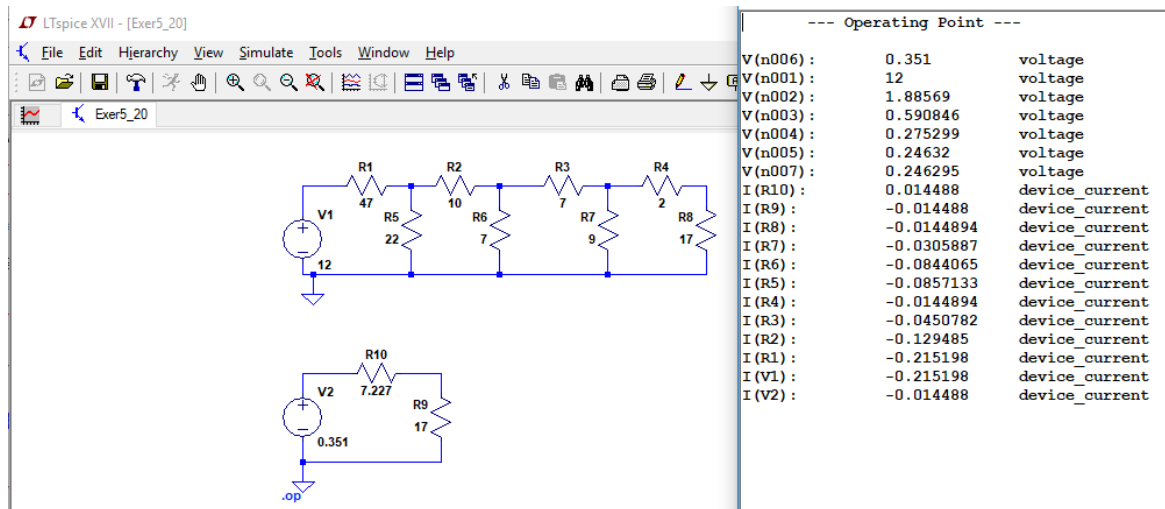
Back to voltage source:  $(0.153)(5.47) = 0.837 \text{ V}$  in series with  $5.47 \Omega$ . Combine with next  $7 \Omega$  to obtain  $12.47 \Omega$ . Back to current source:  $0.837/12.47 = 0.0672 \text{ A}$  in parallel with  $12.47 \Omega$  and  $9 \Omega$ .

$12.47 \parallel 9 = 5.227 \Omega$ . Back to voltage source:  $(0.0672)(5.227) = 0.351 \text{ V}$  in series with  $5.227 \Omega$ . Combine with  $2 \Omega$  to yield  $7.227 \Omega$ .

We are left with a  $0.351 \text{ V}$  source in series with  $7.227 \Omega$  and  $17 \Omega$ .

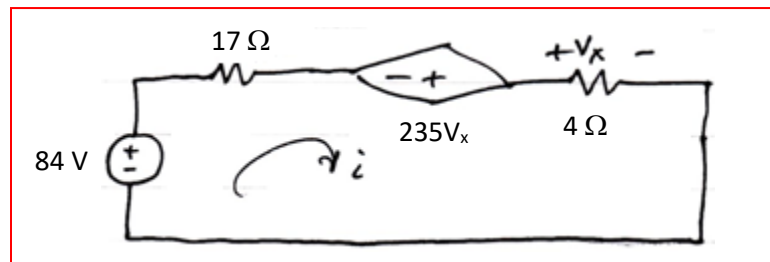
(b) Thus,  $I_x = 0.351/(17 + 7.227) = 14.49 \text{ mA}$  and  $P_{17\Omega} = 17(I_x)^2 = 3.57 \text{ mW}$

- (c) The current through the  $17 \Omega$  resistor in the original circuit is  $14.489 \text{ mA}$ ; in the simplified equivalent (due to small rounding errors) we find  $14.488 \text{ mA}$  – essentially the same, and in agreement with our hand calculations.



21. We combine the 3 A source and 1  $\Omega$  resistor to obtain 3 V in series with 1  $\Omega$  in series with 2  $\Omega$ , all of which is in parallel with 7  $\Omega$ . We transform the 9 A current source into an 81 V source, "+" reference on the bottom, in series with 9  $\Omega$ .

We combine the two 10  $\Omega$  resistors in parallel (5  $\Omega$ ) and the dependent current source to obtain a dependent voltage source, "+" reference on the right, controlled by  $25V_x$ . This source is in series with 5  $\Omega$  and all other transformed components and the 4  $\Omega$  resistor. We may combine the two independent voltage sources and the 1, 2, 5 and 9  $\Omega$  resistors, but must retain the 4  $\Omega$  resistor:



Then defining a clockwise current  $i$  as shown,

$$-84 + 17i - 25V_x + 4i = 0$$

where

$$i = \frac{V_x}{4}$$

Solving,  $V_x = -4.25$  V

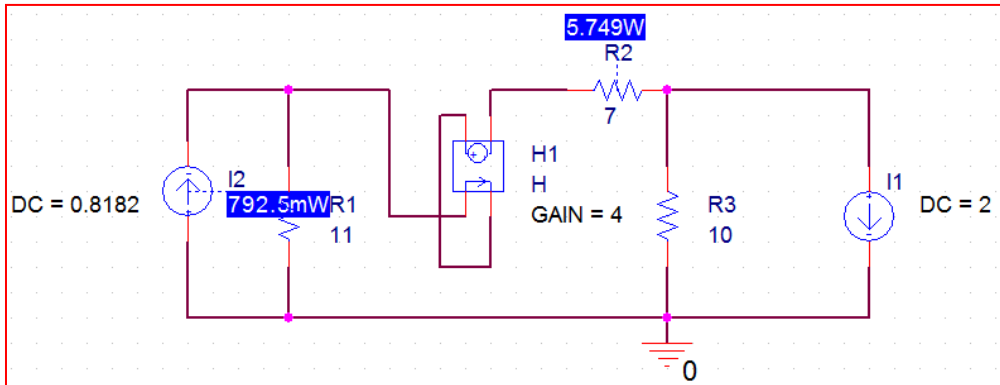
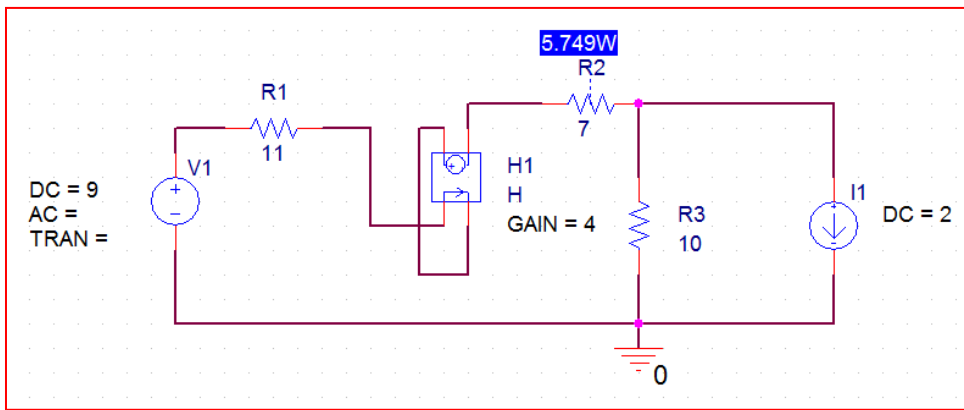
22. (a) Because the controlling current flows through the dependent source as well as the  $7\ \Omega$ , we cannot transform the dependent voltage source into a dependent current source; doing so technically loses  $I_1$ .

Thus, we replace the voltage source and  $11\ \Omega$  resistor with a  $(9/11)\text{ A}$  current source (arrow up) in parallel with an  $11\ \Omega$  resistor.

(b)  $28I_1 - (9/11)(11) - 10(2) + 4I_1 = 0$

Solving,  $I_1 = 29/32\text{ A}$ . Hence,  $P_{7\Omega} = 7(I_1)^2 = 5.75\text{ W}$

(c)



23. Combine  $I_N$  and  $R_N$  to yield a voltage source  $I_N R_N$  in series with  $R_N$ . Combine  $R_C$  and  $I_{CC}$  to yield a voltage source  $I_{CC} R_C$  in series with  $R_C$ .

Then,  $I_B$  is a clockwise mesh current in the lefthand mesh. Defining  $I_C$  as a clockwise mesh current in the righthand mesh,

$$-I_N R_N + V_o + R_E I_B - R_E I_C = 0$$

where  $I_C = -\beta I_B$

so

$$-I_N R_N + V_o + R_E I_B + R_E \beta I_B = 0$$

Solving, 
$$I_B = \frac{I_N R_N - V_o}{(\beta + 1) R_E}$$

24. The independent source may be replaced by a  $(2/6)$  A current source, arrow pointing up, in parallel with  $6 \Omega$ . The dependent voltage source may be replaced by a dependent current source (arrow pointing up) controlled by  $v_3$ . This is in turn in parallel with  $2 \Omega$ .

No further simplification or reduction of components is really possible here.

Choose the bottom node as the reference node. Name the top left node  $v_x$  and the top right node  $v_y$ . Then,

$$\frac{2}{6} - 4v_3 = \frac{v_x}{6} + \frac{v_x - v_y}{3}$$

and

$$v_3 = \frac{v_y - v_x}{3} + \frac{v_y}{2}$$

Since  $v_3 = v_x - v_y$ , we can solve to obtain  $v_3 = 67.42 \text{ mV}$

25. (a)  $V_{th} = 9(3/5) = 27/5 = 5.4 \text{ V}$

$$R_{th} = 1 + 2 \parallel 3 = 2.2 \Omega$$

(b) By voltage division,  $V_L = V_{th} (R_L / (R_L + R_{th}))$

So:

$R_L$	$V_L$
1 $\Omega$	1.688 V
3.5 $\Omega$	3.313 V
6.257 $\Omega$	3.995 V
9.8 $\Omega$	4.410 V

26. (a) Remove  $R_L$ ; Short the 9 V source.

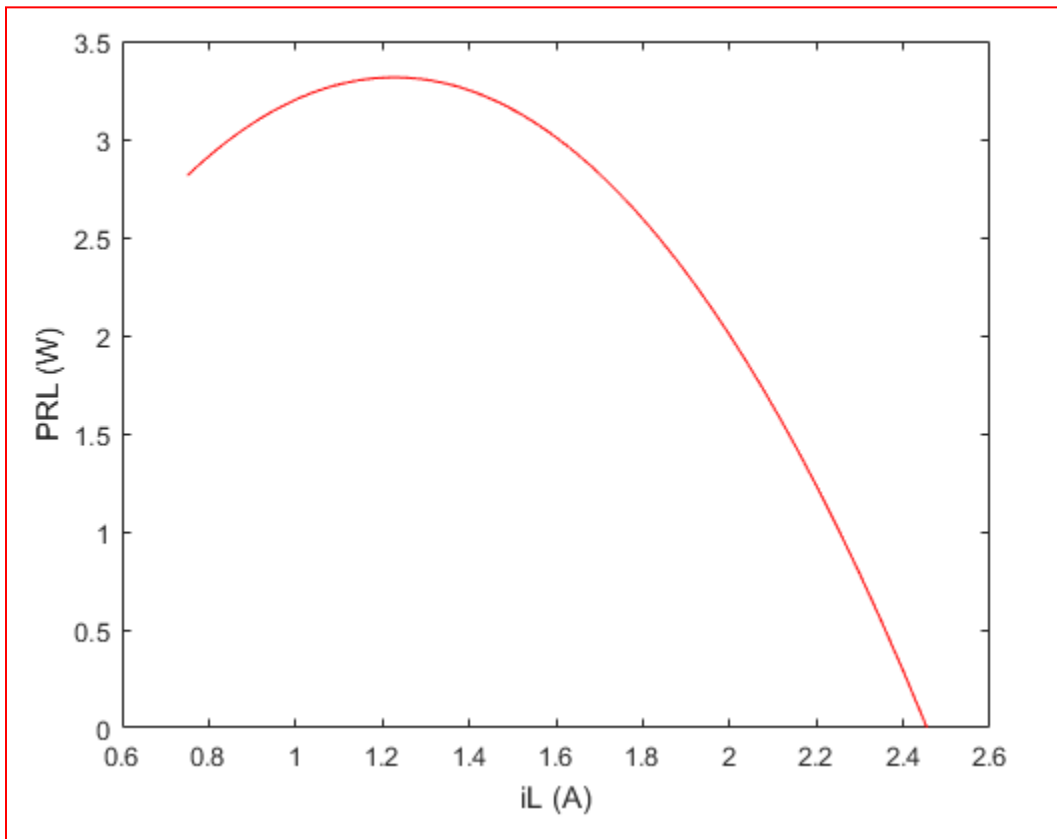
$$R_{th} \text{ seen looking into the terminals} = 1 + 3 \parallel 2 = 2.2 \Omega$$

Returning to the active circuit with  $R_L$  removed,

$$V_{oc} = 9(3)(3 + 2) = 5.4 \text{ V} = V_{th}$$

$$\text{Hence, } I_N = V_{th}/R_{th} = 2.455 \text{ A.}$$

$$(b) P_{R_L} = i_{R_L}^2 R_L = \left( I_N \frac{R_{Th}}{R_L + R_{Th}} \right)^2 R_L \quad \text{and} \quad i_{R_L} = \left( I_N \frac{R_{Th}}{R_L + R_{Th}} \right)$$



(c) This is easier on a plot of power versus load resistance, where it can be seen that maximum power is delivered to  $R = R_{Th} = 2.2 \Omega$ . This corresponds to  $i_L = 1.23 \text{ A}$  on the graph above.

27. (a) By inspection,  $i_N = 5 \text{ A}$ .

Removing the source and  $R_L$  and looking in,  $R_{Th} = 10 \parallel 10 = 5 \Omega$ .

(b)  $V_{Th} = i_N R_{Th} = (5)(5) = 25 \text{ V}$ ,  $R_{Th} = 5 \Omega$ .

(c)

$R_L (\Omega)$	$i_L (\text{A})$	$P_L (\text{W})$
0	5.00	0
1	4.17	17.4
2	3.57	25.5
5	2.50	31.3
10	1.67	27.9

28. (a)  $1.1 \text{ k} \parallel 2.3 \text{ k} = 744 \ \Omega$ ;  $2.5 \text{ k} \parallel 744 = 573.4 \ \Omega$

$$0 = \frac{v_1 - 4.2}{1800} + \frac{v_1}{2500} + \frac{v_1 - v_{oc}}{744} \quad [1]$$

$$0 = \frac{v_{oc}}{2500} + \frac{v_{oc} - v_1}{744} \quad [2]$$

Solving,  $v_{oc} = 1.423 \text{ V} = v_{Th}$

$$i_{sc} = \left( \frac{4.2}{1800} \right) \frac{744^{-1}}{1800^{-1} + 2500^{-1} + 744^{-1}} = 1.364 \text{ mA}$$

Thus,  $R_{th} = v_{oc}/i_{sc} = 1.04 \text{ k}\Omega$

(b)  $P_{4.7k} = (4700)[v_{oc}/(R_{Th} + 4700)]^2 = 289 \ \mu\text{W}$

29. (a)  $1.1 \text{ k} \parallel 2.3 \text{ k} = 744 \ \Omega$ ;  $2.5 \text{ k} \parallel 744 = 573.4 \ \Omega$

$$0 = \frac{v_1 - 4.2}{1800} + \frac{v_1}{2500} + \frac{v_1 - v_{oc}}{744} \quad [1]$$

$$0 = \frac{v_{oc}}{2500} + \frac{v_{oc} - v_1}{744} \quad [2]$$

Solving,  $v_{oc} = 1.423 \text{ V} = v_{th}$

$$i_{sc} = \left( \frac{4.2}{1800} \right) \frac{744^{-1}}{1800^{-1} + 2500^{-1} + 744^{-1}} = 1.364 \text{ mA} = i_N$$

Thus,  $R_{th} = v_{oc}/i_{sc} = 1.04 \text{ k}\Omega$

(b)  $P_{1.7k} = (1700)[v_{oc}/(R_{th} + 1700)]^2 = 459 \ \mu\text{W}$

30. (a) Define three clockwise mesh currents  $i_1$ ,  $i_2$  and  $i_3$ , respectively in the three meshes, beginning on the left. Short the open terminals together. Then, create a supermesh:

$$-0.7 + 45i_1 + 122i_1 - 122i_2 = 0 \quad [1]$$

$$-122i_1 + (122 + 75)i_2 + 220i_3 = 0 \quad [2]$$

$$i_3 - i_2 = 0.3 \quad [3]$$

Solving,  $i_{sc} = i_3 = 100.3 \text{ mA}$

Short the voltage source, open circuit the current source, and look into the open terminals:

$$R_{th} = 220 + 75 + 45 \parallel 122 = 328 \Omega$$

$$\text{Thus, } V_{th} = R_{th}(i_{sc}) = 32.8 \text{ V}$$

$$(b) P_{100\Omega} = (100)[V_{th}/(100 + R_{th})]^2 = 587.3 \text{ mW}$$

(c) Only the second mesh equations needs to be modified:

$$-122i_1 + (122 + 75)i_2 + 220i_3 + 100i_3 = 0 \quad [2']$$

$$\text{Solving, } i_3 = 76.83 \text{ mA} \quad \text{and so } P_{100\Omega} = (100)(i_3)^2 = 590 \text{ mW}$$

31. Define nodal voltages  $V_1$  at the top of the  $3\ \Omega$  resistor and  $V_{oc}$  at the open terminals (the bottom node is our reference node).

Then,

$$-1.5 = \frac{V_1 - 5}{2} + \frac{V_1}{3} + \frac{V_1 - V_{oc}}{5} \quad [1]$$

$$0 = \frac{V_{oc} - V_1}{5} + \frac{V_{oc}}{2} \quad [2]$$

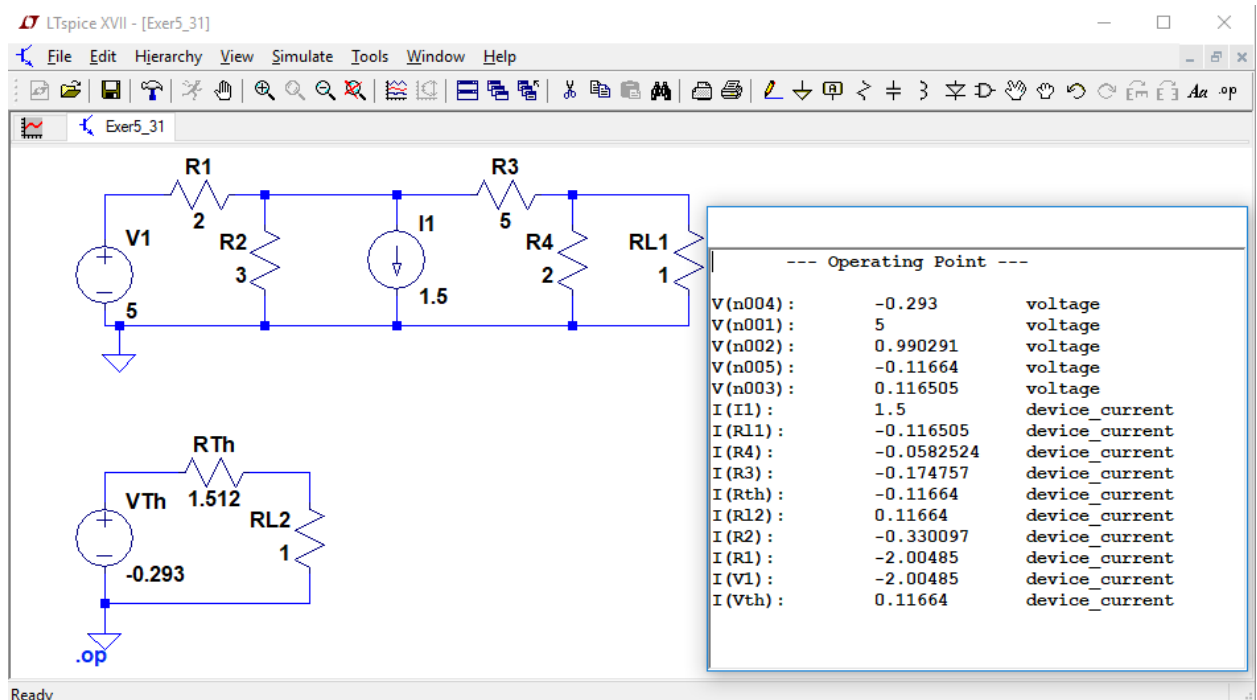
Solving,  $V_{oc} = -0.293\ \text{V} = V_{Th}$

Looking into the terminals of the inactive network,  $R_{Th} = [2||3 + 5] || 2 = 1.512\ \Omega$ .

$$(b) \ i_{1\Omega} = \frac{V_{Th}}{R_{Th} + 1} = -\frac{0.293}{1.512 + 1} = -117\ \text{mA}$$

$$P_{1\Omega} = (i_{1\Omega})^2 (1) = 13.7\ \text{mW}$$

(c) The current through the  $1\ \Omega$  resistor in the original circuit simulates as  $0.1165\ \text{A}$ , and in the Thévenin equivalent circuit as  $0.1166\ \text{A}$ , both in agreement with our hand calculations.



32. Define three clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$  starting on the left.

Then

$$2 + 2i_1 + 6i_2 - i_3 = 0 \quad [1]$$

$$-i_2 + 4i_3 + 4 = 0 \quad [2]$$

$$i_2 - i_1 = 2 \quad [3]$$

Solving,

$$i_2 = 129 \text{ mA. Hence, } v_{oc} = v_x = 5i_2 = \boxed{645.2 \text{ mV}}$$

Next, short the voltage sources and open circuit the current source. Then,

$$R_{th} = 5 \parallel (2 + 3 \parallel 1) = \boxed{1.774 \Omega}$$

33. (a) Employ nodal analysis:

$$2 = \frac{v_1 + 2}{2} + \frac{v_1 - v_2}{5} \quad [1]$$

$$0 = \frac{v_2 - v_1}{5} + v_2 + \frac{v_2 - 4}{3} \quad [2]$$

Solving,  $v_1 = 54/31$  V and  $v_2 = 34/31$  V. Thus,  $V_{TH} = v_X = v_1 - v_2 = 645.2$  mV

By inspection,  $R_{TH} = 5 \parallel [2 + 1 \parallel 3] = 1.774 \Omega$

Thus,  $i_N = v_{TH}/R_{TH} = 363.7$  mA and  $R_N = R_{TH} = 1.774 \Omega$

(b)  $i_{load} = (0.3637)(1.774)/(5 + 1.774) = 95.25$  mA

$$P_{load} = 5(i_{load})^2 = 45.36$$
 mW

(c) 363.7 mA

34. (a) Define three clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$ , respectively, starting on the left, in addition to  $i_{sc}$  which flows through the shorted leads once  $R_L$  is removed.

By inspection,  $i_1 = 0.3$  A, and  $i_{sc} = i_3$  since the  $6$  k $\Omega$  resistor is shorted here.

$$\text{Then, } -7000(0.3) + (13 \times 10^3)i_2 - 1000i_3 + 2.5 = 0 \quad [1]$$

$$-2.5 + 1000i_3 - 1000i_2 = 0 \quad [2]$$

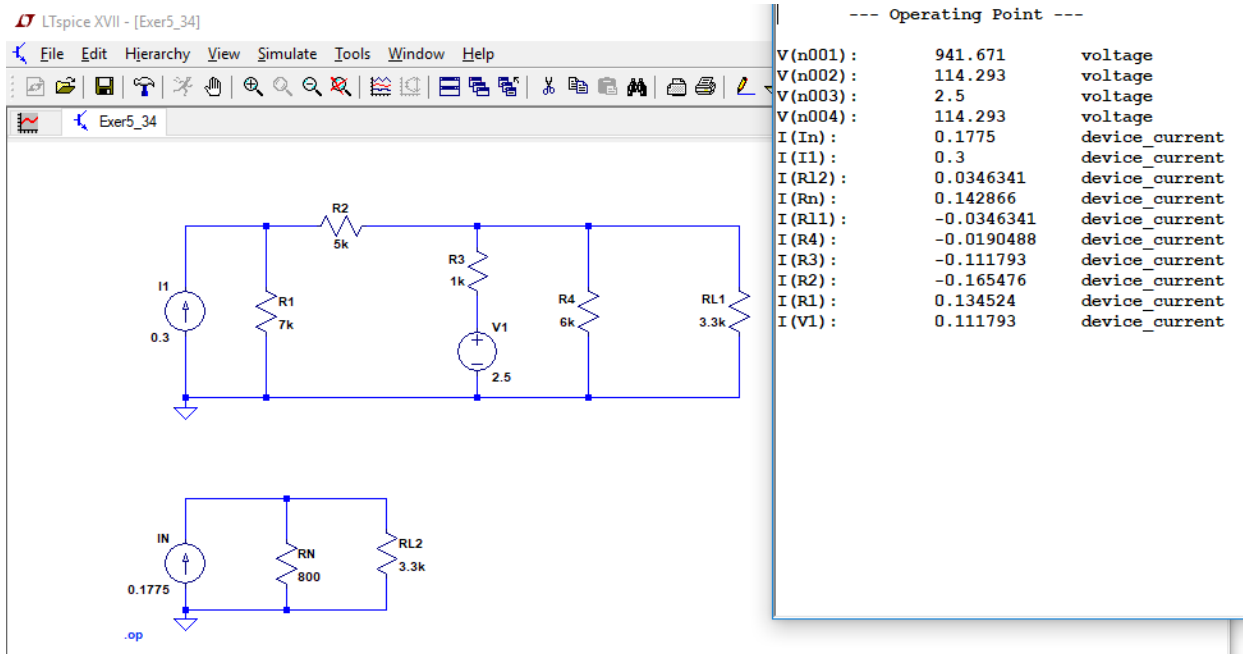
$$\text{Solving, } i_{sc} = i_3 = \boxed{177.5 \text{ mA}}$$

Looking into the open terminals with the sources zeroed,

$$R_{th} = 6000 \parallel 10000 \parallel 120000 = \boxed{800 \Omega}$$

$$(b) i_{R_L} = i_{sc} \frac{R_{TH}}{R_L + R_{TH}} = \boxed{34.63 \text{ mA}}$$

- (c) The two currents agree with each other, and with our hand calculation. (The sign difference is an artifact of how  $R_{L2}$  was placed. Rotating it by  $180$  degrees will yield the same sign.)



35. (a) We select the bottom node as the reference node. The top left node is then  $-2$  V by inspection; the next node is named  $v_1$ , the next  $v_2$ , and the far right node is  $v_{oc}$ .

$$0 = \frac{v_1 + 2}{10} + \frac{v_1}{7} + \frac{v_1 - v_2}{20} \quad [1]$$

$$0 = \frac{v_2 - v_1}{20} + \frac{v_2}{7} \quad [2]$$

Solving,

$$v_2 = v_{oc} = -185.3 \text{ mV}$$

Next, we short the output terminals and compute the short circuit current. Naming the three clockwise mesh currents  $i_1$ ,  $i_2$  and  $i_{sc}$ , respectively, beginning at the left,

$$2 + 17i_1 - 7i_2 = 0 \quad [1]$$

$$-7i_1 + 34i_2 - 7i_{sc} = 0 \quad [2]$$

$$-7i_2 + 37i_{sc} = 0 \quad [3]$$

Solving,  $i_{sc} = -5.2295$  mA.

Hence

$$R_{TH} = \frac{v_{oc}}{i_{sc}} = \boxed{35.43 \ \Omega}$$

(b) Connecting a 1 A source to the dead network, we can simplify by inspection, but performing nodal analysis anyway:

$$0 = \frac{v_1}{10} + \frac{v_1}{7} + \frac{v_1 - v_2}{20} \quad [1]$$

$$0 = \frac{v_2 - v_1}{20} + \frac{v_2}{7} + \frac{v_2 - v_{test}}{30} \quad [2]$$

$$1 = \frac{v_{test} - v_2}{30} \quad [3]$$

Solving,  $v_{test} = 35.43$  V hence  $R_{TH} = 35.43/1 = \boxed{35.43 \ \Omega}$

(c) Connecting a 1 A source, we can write three mesh equations after defining clockwise mesh currents:

$$0 = 17i_1 - 7i_2 \quad [1]$$

$$0 = -7i_1 + 34i_2 - 7i_3 \quad [2]$$

$$-1 = -7i_2 + 37i_3 \quad [3]$$

Solving,

$$i_3 = -28.23 \text{ mA. Thus, } R_{TH} = 1/(-i_3) = \boxed{35.42 \ \Omega}$$

36. (a) We can ignore the  $3\ \Omega$  resistor to determine  $v_{oc}$ . Then,  $i_{4\Omega} = (1)(2)/(2 + 5) = 2/7\ \text{A}$ .  
Hence,  $v_{oc} = 4i_{4\Omega} = 1.143\ \text{V}$

$i_{sc}$ : A source transformation is helpful here, yielding  $2\ \text{V}$  in series with  $2\ \Omega$ . Then noting that  $3\ \parallel\ 4 = 1.714\ \Omega$ ,  $V_{3\Omega} = 2(1.714)/(3 + 1.714) = 0.7272\ \text{V}$

Hence,

$$i_{sc} = v_{3\Omega}/3 = 242.4\ \text{mA}$$

Consequently,  $R_{TH} = v_{oc}/i_{sc} = 4.715\ \Omega$

- (b) Connect the  $1\ \text{A}$  source as instructed to the dead network, and define  $v_x$  across the source. Then  $v_x = (1)(3 + 4\ \parallel\ 3) = 4.714\ \text{V}$ .

Hence,

$$R_{TH} = 4.714\ \Omega$$

- (c) Connect the  $1\ \text{V}$  source to the dead network as instructed, and define  $i_x$  flowing out of the source. Then,  $i_x = [3 + 3\ \parallel\ 4]^{-1} = 1/4.714\ \text{A}$ .

Consequently,  $R_{TH} = 1/i_x = 4.714\ \Omega$

37. (a) Choose the bottom node as the reference, and name the top right node  $v_{oc}$ . Then,

$$-10 = \frac{v_{oc} - 20}{9} + \frac{v_{oc}}{4} + \frac{v_{oc}}{2}$$

Solving,

$$v_{oc} = -9.03 \text{ V.}$$

$$I_{sc} = 20/9 - 10 = -7.78 \text{ A}$$

$$\text{Hence, } R_{TH} = v_{oc}/i_{sc} = 1.16 \Omega$$

$$\text{(b) By inspection, } R_{TH} = 2 \parallel 4 \parallel 9 = 1.16 \Omega$$

- (c) Connect current source  $i_x$  such that the arrow points into the top open terminal. Define  $v_x$  across the current source such that the '+' reference is at the head of the current source arrow.

$$i_x = \frac{v_x}{9} + \frac{v_x}{4} + \frac{v_x}{2}$$

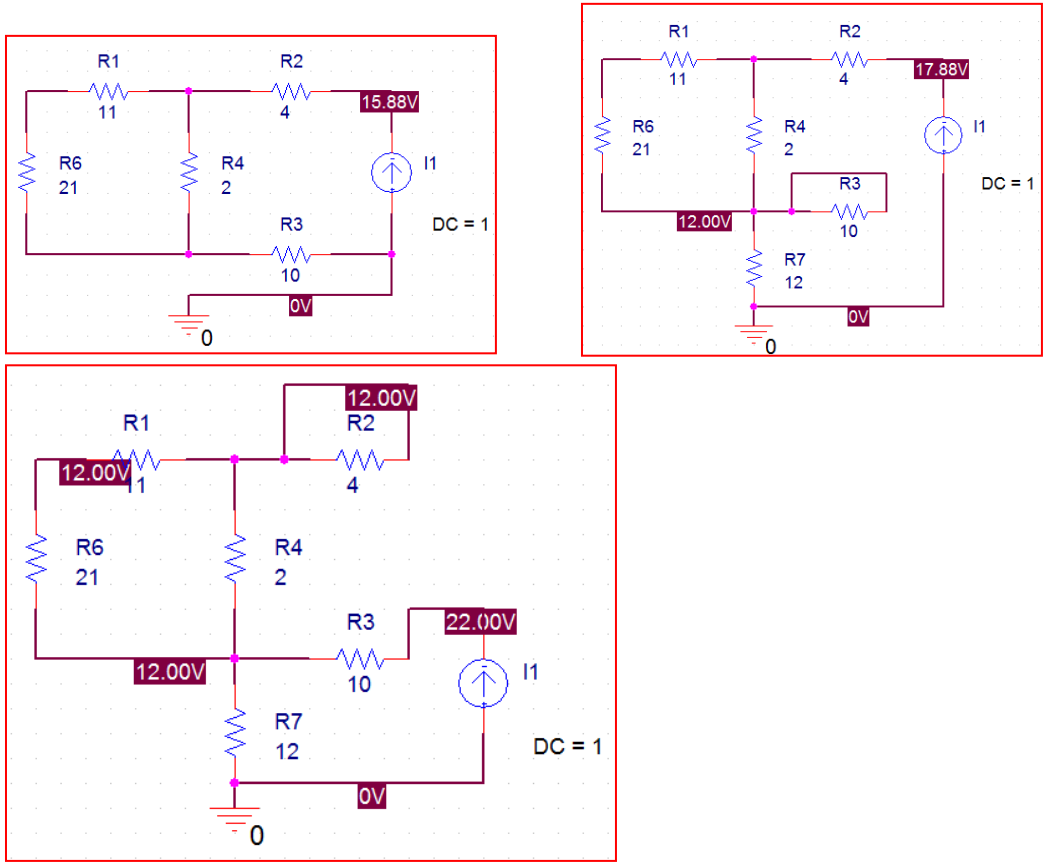
$$\text{Hence, } R_{Th} = \frac{v_x}{i_x} = 1.16 \Omega$$

38. (a) Between terminals *a* and *b*,  $R_{TH} = 4 + (11 + 21) \parallel 2 + 10 = 15.88 \Omega$

(b) Between terminals *a* and *c*,  $R_{TH} = 4 + (11 + 21) \parallel 2 + 12 = 17.88 \Omega$

(c) Between terminals *b* and *c*,  $R_{TH} = 10 + 12 = 22 \Omega$

(d) Note that the magnitude of  $R_{TH}$  is the same as that of the voltage across the 1 A source in each simulation, respectively.



39. We connect a 1 A source across the open terminals of the dead network, and compute the voltage  $v_x$  which develops across the source.

By nodal analysis,  $1 + 10v_x = v_x/21$ . Solving,  $v_x = 0.1005$  V.

Hence,

$$R_{TH} = v_x/1 = -100.5 \text{ m}\Omega$$

(the dependent source helps us achieve what appears to be a negative resistance!)

40. We short the terminals of the network and compute the short circuit current. To do this, define two clockwise mesh currents (the 1500  $\Omega$  resistor is shorted out).

$$-500i_x + 2i_x + 2500i_2 = 0 \quad [1]$$

$$-i_2 - i_x = 0.7 \quad [2]$$

Solving,  $i_2 = i_{sc} = i_N = -116.3 \text{ mA}$

Next, we zero out the independent source and connect a 1 A test source across the terminals  $a$  and  $b$  such that 1 A flows into  $a$ . Define  $v_{test}$  with the '+' reference at terminal  $a$ , and the cathode at terminal  $b$ . Then,

$$-2i_x + 500i_x + 2500i_x + 1500i_x - 1500(1) = 0$$

so

$$i_x = 0.333 \text{ A}$$

Solving,  $v_{test} = 1500 - i_x$  so  $R_{TH} = v_{test}/1 = 1000 \Omega$

41. To find  $i_{sc}$ , short circuit the  $10\text{ k}\Omega$  resistor. This sets  $v_1 = 0$ , effectively open circuiting the dependent source. Hence,

$$i_{sc} = 1/10 \times 10^3 = 0.1\text{ mA}$$

To find  $v_{oc}$ , we define a clockwise mesh current  $i_2$  such that it flows into the '+' reference of the  $1\text{ V}$  source. Then,

$$-v_1 + 1 + 20 \times 10^3 (i_2 - 2 \times 10^{-4} v_1) = 0$$

where

$$v_1 = -10 \times 10^3 i_2$$

Hence,

$$v_1 = 0.143\text{ V} = v_{oc}$$

Thus,  $I_N = 100\text{ }\mu\text{A}$ , arrow pointing down, and  $R_{TH} = v_{oc}/i_{sc} = 1.43\text{ k}\Omega$ .

42. Rotate the diagram 90° clockwise. Connect a 1 A source between  $a$  and  $b$  such that the head of the arrow points to terminal  $a$ . Define a clockwise mesh current  $i_2$  flowing into the '+' reference of the dependent voltage source. Then,

$$11(i_2 - 0.11v_{ab}) + 20i_2 + 0.5v_{ab} + 15(i_2 + 1) = 0$$

where

$$v_{ab} = 15(i_2 + 1)$$

Solving,  $v_{ab} = 13.15$  V.

Hence,  $R_{Th} = v_{ab}/1 = 13.15 \Omega$ .

Since there is no independent source in the network, this represents both the Thévenin and Norton equivalent.

43. Connect a 1 A source to the open terminals, and select the bottom node as the reference terminal. Define  $v_1$  at the top of the 1 A source. Then

$$1 = v_1/10^6 \quad \text{so } v_1 = 10^6 \text{ V and } R_{\text{Th}} = v_1/1 = 1 \text{ M}\Omega$$

44. Disconnect the two elements left of the dashed line. We note our solution will be  $2\text{ M}\Omega$  in parallel with another effective resistance. To simplify, remove the  $2\text{ M}\Omega$  resistor. Then, apply a  $1\text{ A}$  test source to the open terminals and define  $v_x$  across the  $1\text{ A}$  source such that the “+” reference is at the arrow head of the source. Select the bottom node as the reference and define  $v_2$ .

$R_{\text{eq}} = v_x/1$ , so we need an expression for  $v_x$ .

$$v_x = v_\pi + v_2 = r_\pi(1) + v_2 = r_\pi + (1 + 0.02r_\pi)(1000||2000)$$

or

$$v_x = r_\pi + 667(1 + 0.02r_\pi)$$

$$\text{Hence, } R_{\text{th}} = \frac{2 \times 10^6 [r_\pi + 667(1 + 0.02r_\pi)]}{2 \times 10^6 + r_\pi + 667(1 + 0.02r_\pi)}$$

45. First, we determine  $v_{TH}$  by employing nodal analysis:

$$0 = \frac{-v_d - v_{in}}{R_1} + \frac{-v_d}{R_i} + \frac{-v_d - v_{out}}{R_f} \quad [1]$$

$$0 = \frac{v_{out} - Av_d}{R_o} + \frac{v_{out} + v_d}{R_f} \quad [2]$$

Solving,

$$v_{out} = \frac{(R_o - AR_f)R_i}{R_1R_f + R_1R_i + R_1R_o + R_fR_i + R_iR_o + AR_1R_i} v_{in}$$

We now find  $R_{TH}$  by injecting 1 A of current into the dead network and determining the voltage which develops:

$$0 = \frac{-v_d}{R_1} + \frac{-v_d - v_{out}}{R_f} - \frac{v_d}{R_i} \quad [1] \text{ and}$$

$$1 = \frac{v_{out} + v_d}{R_f} + \frac{v_{out} - Av_d}{R_o} \quad [2]$$

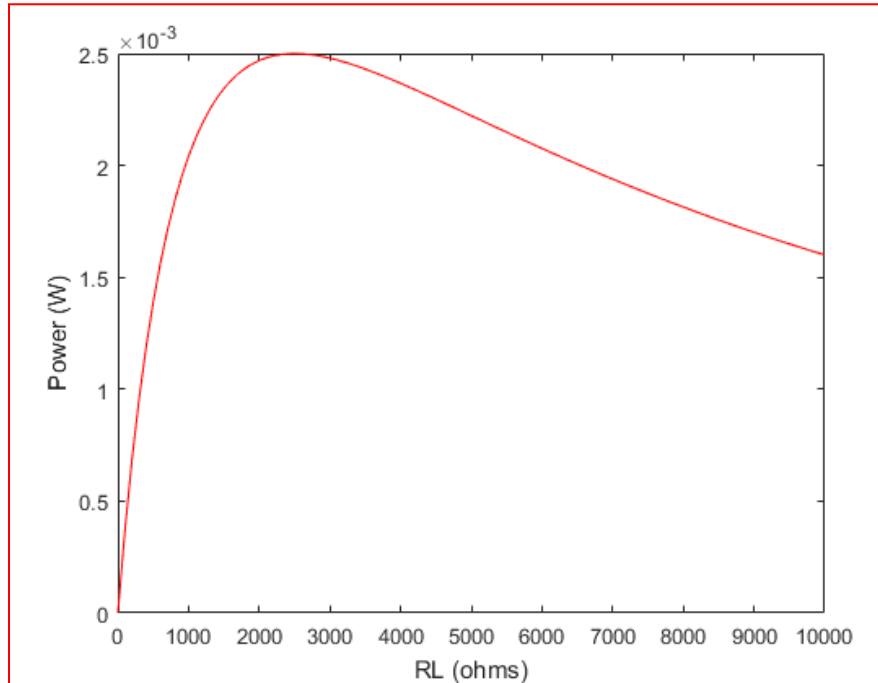
$$\text{Solving, } R_{TH} = \frac{v_{out}}{1} = v_{out} = \frac{R_o(R_iR_f + R_1R_f + R_1R_i)}{R_iR_o + R_1R_o + R_iR_f + R_1R_f + R_1R_i + AR_1R_i}$$

46. (a) Open circuiting the current source, removing the load resistor, and looking back into those terminals, we see  $5\text{ k} \parallel 5\text{ k}$  or

$$R_{Th} = 2.5\text{ k}\Omega$$

$$V_{Th} = V_{oc} = (2 \times 10^{-3})(2.5 \times 10^3) = 5\text{ V}$$

$$(b) P_{R_L} = (i_{R_L})^2 R_L = \left[ \frac{V_{Th}}{R_{Th} + R_L} \right]^2 R_L$$



- (c) When  $R_L = R_{Th} (= 2500\ \Omega)$ .

- (d)

$$P_{\max} = \left[ \frac{V_{Th}}{2R_{Th}} \right]^2 R_{Th} = 2.5\text{ mW}$$

Half would be 1.25 mW, requiring

$$\left[ \frac{V_{Th}}{2R_L} \right]^2 R_L = 1.25\text{ mW}$$

or

$$R_L = 429\ \Omega$$

47. (a) Define a clockwise mesh current  $i$ . Then  $5i + 4 + 2 = 0$  and  $i = -6/5$  A

$$v_{\text{out}} = 2 + 2i = 2 - 12/5 = -400 \text{ mV}$$

By inspection of the dead network,  $R_{\text{TH}} = 2 \parallel 3 = 1.2 \Omega$

(b) Choose  $R_{\text{out}} = R_{\text{TH}} = 1.2 \Omega$

48. (a) A quick source transformation and we have all voltage sources. Then, remove  $R_{out}$  and short the open terminals. Mesh analysis yields

$$+4 + 1000i_1 + 2000i_1 + 2 - 2000i_2 = 0 \quad [1]$$

$$-2000i_1 + 2000i_2 - 2 + 3 = 0 \quad [2]$$

Solving,

$$i_N = i_2 = \boxed{-7.5 \text{ mA}}$$

Next, zero out all sources, remove  $R_{out}$ , and look into the open terminals.

$$R_{Th} = 1000 \parallel 2000 = \boxed{667 \Omega}$$

- (b) Maximum power is obtained for  $R_{out} = R_{Th} = \boxed{667 \Omega}$

49. Yes, it would theoretically result in maximum power transfer. Since we're charged for the energy we use (power multiplied by time), this would cost the consumer a fortune. In reality, we don't want all the power the utility can provide – only the amount we need!

50. We need only  $R_{Th}$ . Setting all sources to zero, removing  $R_L$ , and looking into the terminals,

$$R_{Th} = 5 \parallel 2 \parallel 3 = 968 \text{ m}\Omega$$

Setting  $R_L = R_{Th} = 968 \text{ m}\Omega$  achieves maximum power delivery.

However, we note that the two sources cancel, such that  $V_{Th} = 0$ . So, technically,  $R_L$  will absorb 0 W regardless of its value, to be pedantic.

51. We select the bottom node as reference, and then define two nodal voltages  $V_a$  and  $V_b$ .  
 $V_a = (4)(5) = 20 \text{ V}$  and  $V_b = (-2)(2) = -4 \text{ V}$

$$\text{Thus, } V_{ab} = V_{oc} = V_{Th} = V_a - V_b = 20 - (-4) = 24 \text{ V}$$

$$\text{By inspection, } R_{Th} = 5 + 2 = 7 \Omega$$

$$\text{(b) } I_N = V_{Th}/R_{Th} = 24/7 \text{ A}$$

$$\text{(c) Selecting } R_L = R_{Th} = 7 \Omega$$

52. (a) Define clockwise mesh current  $i$ . Then  
 $-0.1v_2 + 2i - 5 + 7i + 3.3i = 0$  where  $v_2 = 3.3i$

Hence,

$$-0.1(3.3i) + 12.3i = 5$$

Solving,

$$i = 417.7 \text{ mA} \text{ and so } v_2 = 1.378 \text{ V} = v_{\text{TH}}$$

Consequently,  $P = (v_2)^2/3.3 = 575.4 \text{ mW}$

- (b) Find  $R_{\text{TH}}$  by connecting a 1 A source across the open terminals.

$$1 = \frac{v_2 - 0.1v_2}{9}. \text{ Solving, } v_2 = 10 \text{ V.}$$

$$\text{Thus, } R_{\text{TH}} = v_2/1 = 10 \Omega$$

Hence, replace the  $3.3 \Omega$  resistor in the original circuit with  $10 \Omega$ .

53. We connect a 1 A source across the open terminals and define  $v_{\text{test}}$  across the source such that its “+” reference corresponds to the head of the current source arrow. Then, after defining nodal voltages  $v_1$  and  $v_2$  at the top left and top right nodes, respectively,

$$1 = \frac{v_1}{5} + \frac{v_1}{8} \quad \text{and so } v_1 = 3.077 \text{ V}$$

$$-1 + 0.2v_1 = \frac{v_2}{10} \quad \text{so } v_2 = -3.846 \text{ V}$$

By KVL,  $v_{\text{test}} = v_1 - v_2 = 6.923 \text{ V}$  so  $R_{\text{TH}} = 6.923 \Omega$ . We select this value for  $R_L$  to ensure maximum power transfer.

54. We zero out the current source and connect 1 V to terminals  $a$  and  $b$ . This allows us to know a priori what the dependent source values are, since they are controlled by that voltage. Using iterative source transformation,

$$(0.1)(100 \parallel 50) = 3.33 \text{ V in series with } 33.3 \Omega$$

Combine the 3.33 V and 2 V sources, and the 33.3  $\Omega$  and 20  $\Omega$  resistors.

$$\text{Thus, } \frac{5.33}{53.3} + I_{1V} = \frac{1}{53.3} + \frac{1}{10} \text{ and the current flowing from our 1 V test source} = 0.01876.$$

$$R_{Th} = 1/0.01876 = 53.3 \Omega$$

Thus, select 53.3  $\Omega$  to obtain maximum power transfer.

55. We note that the equations which describe the two equivalent circuits are already developed and provided as Eqs. 23-24 and Eqs. 25-26, respectively. Equating terms most directly results in the equations for  $R_1$ ,  $R_2$  and  $R_3$ .

The next step is to divide those equations to find the following ratios:

$$\frac{R_1}{R_2} = \frac{R_A}{R_C}; \quad \frac{R_1}{R_3} = \frac{R_B}{R_C} \quad \text{and} \quad \frac{R_2}{R_3} = \frac{R_B}{R_A}.$$

These three equations yield two equations for  $R_A$ , two for  $R_B$  and two for  $R_C$ , which may be equated (respectively) to obtain:

$$\frac{R_3}{R_2} R_B - \frac{R_1}{R_2} R_C = 0$$

$$\frac{R_2}{R_3} R_A - \frac{R_1}{R_3} R_C = 0$$

$$\frac{R_2}{R_1} R_A - \frac{R_3}{R_1} R_B = 0$$

Solving, we find that

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}, \quad R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}, \quad R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

56. For the first circuit, we compute  $\Sigma = 33 + 17 + 22 = 71 \Omega$ .

Then,

$$R_1 = (33)(17)/\Sigma = 7.901 \Omega$$

$$R_2 = (17)(21)/\Sigma = 5.028 \Omega$$

$$R_3 = (21)(33)/\Sigma = 9.761 \Omega$$

For the second circuit,  $\Sigma = 1.1 + 4.7 + 2.1 = 7.9 \text{ k}\Omega$

Then,

$$R_1 = (1.1)(4.7)/7.9 = 654.4 \Omega$$

$$R_2 = (4.7)(2.1)/7.9 = 1.249 \text{ k}\Omega$$

$$R_3 = (2.1)(1.1)/7.9 = 292.4 \Omega$$

57.

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

For the circuit on the left,  $R_1 = 33 \Omega$ ,  $R_2 = 21 \Omega$  and  $R_3 = 17 \Omega$ .

$R_1 R_2 + R_2 R_3 + R_3 R_1 = 1611$  so

$$R_A = 1611/21 = 76.71 \Omega$$

$$R_B = 1611/17 = 94.76 \Omega$$

$$R_C = 1611/33 = 48.82 \Omega$$

For the circuit on the right,  $R_1 = 1300 \Omega$ ,  $R_2 = 2100 \Omega$  and  $R_3 = 4700 \Omega$ .

$R_1 R_2 + R_2 R_3 + R_3 R_1 = 1.871 \times 10^7$  so

$$R_A = 1.871 \times 10^7 / 2100 = 8.910 \text{ k}\Omega$$

$$R_B = 1.871 \times 10^7 / 4700 = 3.981 \text{ k}\Omega$$

$$R_C = 1.871 \times 10^7 / 1300 = 14.39 \text{ k}\Omega$$

58. We begin by converting the bottom section to a T network ( $R_1$ ,  $R_2$ ,  $R_3$ )

$$\Sigma = 2 + 3 + R = 5 + R$$

$$R_1 = 2R/\Sigma = 2R/(5 + R)$$

$$R_2 = 3R/\Sigma = 3R/(5 + R)$$

$$R_3 = (3)(2)/\Sigma = 6/(5 + R)$$

We now have  $30 \Omega$  in series with  $R_1$ ,  $10 \Omega$  in series with  $R_2$ . Those branches are in parallel. The total is in series with  $R_3$ .

The new network then is equivalent to

$$\begin{aligned} (30 + R_1) \parallel (10 + R_2) + R_3 &= \left(30 + \frac{2R}{5 + R}\right) \parallel \left(10 + \frac{3R}{5 + R}\right) + \frac{6}{5 + R} \\ &= \frac{300(5 + R) + 110R + \frac{6R^2}{5 + R}}{40(5 + R) + 5R} + \frac{6}{5 + R} = 9 \end{aligned}$$

Solving,  $R = 5.5 \Omega$  (rounded)

59.  $R_A = 42 \Omega$ ;  $R_B = 200 \Omega$ ;  $R_C = 68 \Omega$ .

Then  $R_1 = 27.10 \Omega$ ,  $R_2 = 43.87 \Omega$ , and  $R_3 = 9.213 \Omega$

The new network is then  $(100 + 27.1) \parallel (R + 43.87) + 9.213 = 70.6 \Omega$

Solving,  $R = 74.86 \Omega$

60. Define  $R_x = R \parallel R_B$   
 $R_A = R_B = R_C = 3R^2/R = 3R$

Thus,  $R_x = (3R)(R)/(3R + R) = (3/4)R$

Define  $R_{11} = (3R)(0.75R)/(3R + 0.75R + 3R) = R/3$

$R_{22} = (0.75R)(3R)/(3R + 0.75R + 3R) = R/3$

$R_{33} = (3R)(3R)/(3R + 0.75R + 3R) = 4R/3$

Combine series resistances then define

$R_{AA} = (R^2 + 4R^2/3 + 4R^2/3) = 11R/3$

$R_{BB} = (R^2 + 4R^2/3 + 4R^2/3) = 11R/4$

$R_{CC} = (R^2 + 4R^2/3 + 4R^2/3)/R = 11R/3$

The equivalent resistance is then  $2[(4R/3) \parallel (11R/3)] \parallel (11R/4) = 2514 \Omega$

61. We can neglect the 110  $\Omega$  resistor (no current flow).

Identify the 11, 23 and 31  $\Omega$  as  $R_1$ ,  $R_2$ ,  $R_3$  and convert to a  $\Delta$  network with

$$R_{A1} = 118.8 \Omega$$

$$R_{B1} = 42.2 \Omega$$

$$R_{C1} = 56.8 \Omega$$

Identify the 55, 46, and 61  $\Omega$  as  $R_1$ ,  $R_2$ ,  $R_3$  and convert to a  $\Delta$  network with

$$R_{A2} = 188.9 \Omega$$

$$R_{B2} = 142.5 \Omega$$

$$R_{C2} = 158.0 \Omega$$

$$\text{Then } R_w = R_{A1} \parallel 31 = 24.6 \Omega$$

$$R_x = R_{B1} \parallel R_{B2} = 32.6 \Omega$$

$$R_y = 25 \parallel R_{A2} = 22.1 \Omega$$

We are left with one last transformation, converting  $R_y$ ,  $R_{C1}$  and  $R_x$  into a  $\Delta$  network:

$$R_A = \frac{R_y R_{C1} + R_{C1} R_x + R_x R_y}{R_{C1}} = \frac{3827}{56.8} = 67.4 \Omega$$

$$R_B = \frac{R_y R_{C1} + R_{C1} R_x + R_x R_y}{R_x} = \frac{3827}{32.6} = 117.4 \Omega$$

$$R_C = \frac{R_y R_{C1} + R_{C1} R_x + R_x R_y}{R_y} = \frac{3827}{22.1} = 173.2 \Omega$$

$$\text{Then, } R_{in} = 63 \parallel R_B \parallel [R_A \parallel R_{C2} + R_C \parallel R_w] = 25.7 \Omega$$

62. We begin by noting that  $(6 + 12) \parallel 20 = 9.474 \Omega$ .

$$\text{Define } \Sigma = R_1R_2 + R_2R_3 + R_3R_1 = (9)(5) + (5)(6) + (6)(0) = 129$$

$$\text{Then } R_A = \Sigma/R_2 = 25.8 \Omega$$

$$R_B = \Sigma/R_3 = 21.5 \Omega$$

$$R_C = \Sigma/R_1 = 14.3 \Omega$$

After this conversion, we have  $R_C \parallel 4 = 3.126 \Omega$ ,  $R_A \parallel 3 = 2.688 \Omega$ .

Now define  $\Sigma = R_{AA} + R_{BB} + R_{CC} = 2.688 + 21.5 + 2.126 = 27.31 \Omega$ .

Then  $R_{11} = 2.116$ ,  $R_{22} = 2.461$  and  $R_{33} = 0.0377$ . This last resistance appears in series with  $10 \Omega$ .

Performing one last conversion,

$$\text{Define } \Sigma = (2.116)((2.461) + (2.461)(10.31) + (10.31)(2.116)) = 52.40$$

$$R_a = \Sigma/2.461 = 21.29 \Omega$$

$$R_b = \Sigma/10.31 = 5.082 \Omega$$

$$R_c = \Sigma/2.116 = 24.76 \Omega$$

By inspection,

$$R_{Th} = R_b \parallel [(R_c \parallel 7) + (R_a \parallel 9.474)] = 3.57 \Omega$$

63. (a) Shorting the 9 V source leaves the 11  $\Omega$  in parallel with 2  $\Omega$ . Name this  $R_A = 1.69 \Omega$ . Then,  $R_B = 22 \Omega$ , and  $R_C = 1 \Omega$ .

Converting this to a T network, with  $R = R_A + R_B + R_C = 24.69 \Omega$ .

$$R_1 = \frac{R_A \cdot R_B}{R} = 1.51 \Omega$$

$$R_2 = \frac{R_B \cdot R_C}{R} = 0.891 \Omega$$

$$R_3 = \frac{R_C \cdot R_A}{R} = 0.0684 \Omega$$

$$\text{Then, } R_{Th} = (12+R_1) \parallel (10+R_2) + R_3 = 6.098 \Omega.$$

Define three nodal voltages on the original circuit after naming the bottom node as the reference node:  $V_1$ ,  $V_2$ ,  $V_{oc}$ .

Then,

$$0 = \frac{V_1 - 9}{11} + \frac{V_1}{2} + \frac{V_1 - V_{oc}}{12} + \frac{V_1 - V_2}{22}$$

$$0 = \frac{V_2}{1} + \frac{V_2 - V_1}{22} + \frac{V_2 - V_{oc}}{10}$$

$$0 = \frac{V_{oc} - V_2}{10} + \frac{V_{oc} - V_1}{12}$$

Solving,  $V_{oc} = -606.7 \text{ mV}$

$$(b) P_{1\Omega} = [0.6067/7.098]^2 (1) = 7.306 \text{ mW}$$

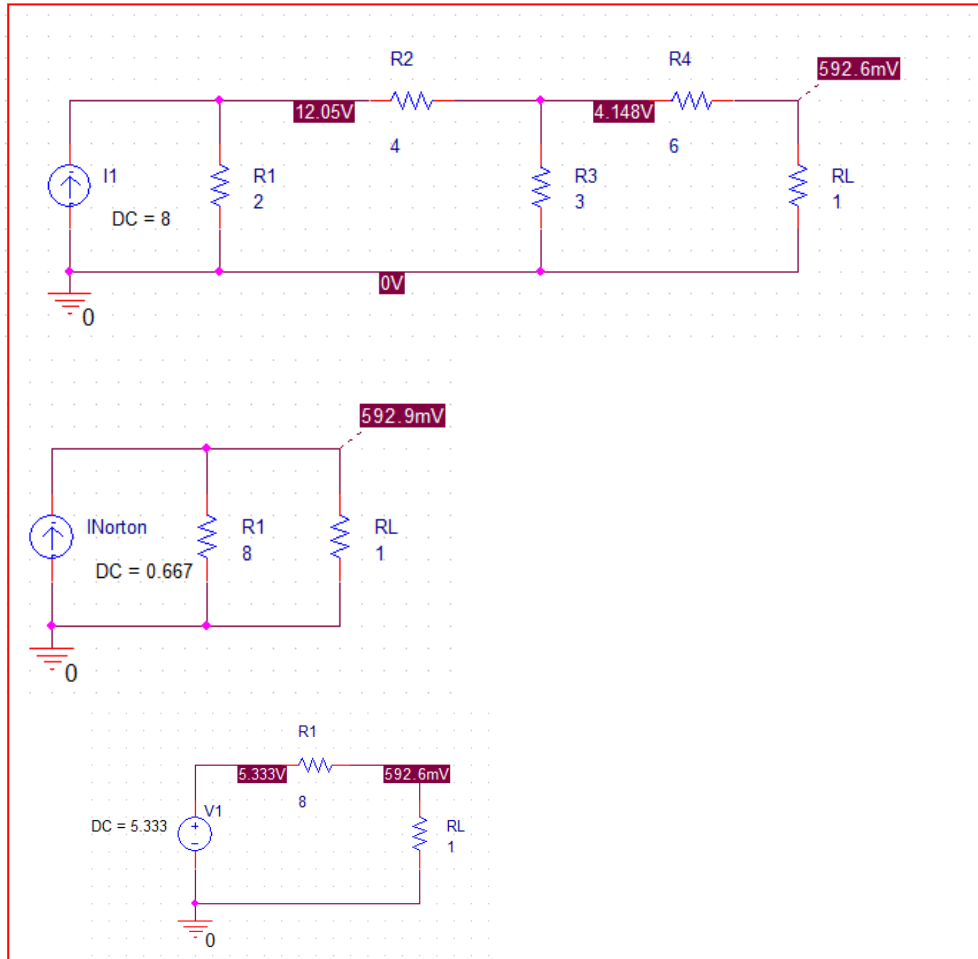
64. (a) By inspection,  $R_{Th} = 6 + 6 \parallel 3 = 8 \Omega$

$$\text{Then, } V_{oc} = 16(3)/(6 + 3) = 5.333 \text{ V}$$

$$I_N = V_{oc}/R_{Th} = 5.333/8 = 666.7 \text{ mA}$$

Thus, the Thévenin equivalent is 5.333 V in series with  $8 \Omega$  and the Norton equivalent is 666.7 mA in parallel with  $8 \Omega$

- (b) The three simulations agree within acceptable rounding error.



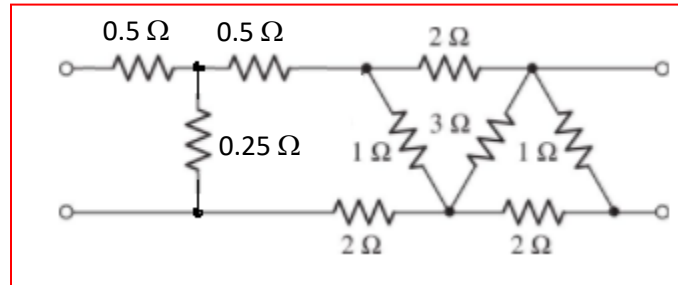
65. (a) We name the resistors (left to right)  $R_A = 1 \Omega$ ,  $R_B = 2 \Omega$ , and  $R_C = 1 \Omega$ . Then,  $R_A + R_B + R_C = 4 \Omega$  and

$$R_1 = \frac{R_A R_B}{4} = 0.5 \Omega$$

$$R_2 = \frac{R_B R_C}{4} = 0.5 \Omega$$

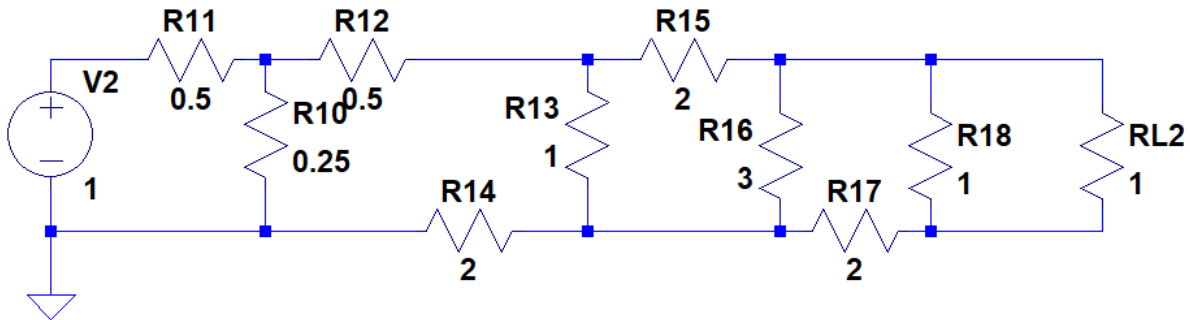
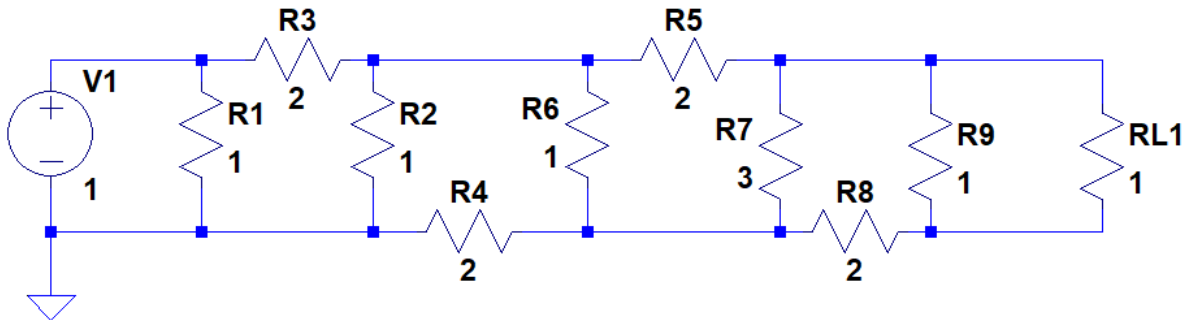
$$R_3 = \frac{R_C R_A}{4} = 0.25 \Omega$$

Thus, our final circuit is



- (b) Using a  $1 \Omega$  load resistor and a  $1 \text{ V}$  test source, we note both sources provide the same current.

I (V2) :	-1.36566	device_current
I (V1) :	-1.36566	device_current



.op

66. The wording points to the need for a Thévenin (Norton) equivalent. Simplifying using  $\Delta$ - $\Pi$  conversion, note  $1\text{ k} \parallel 7\text{ k} = 875\ \Omega$ ;  $10\text{ k} \parallel 2.2\text{ k} = 1.803\text{ k}\Omega$

$$R_1 = (10)(4)/19 = 2.105\text{ k}\Omega$$

$$R_2 = (4)(5)/19 = 1.053\text{ k}\Omega$$

$$R_3 = (5)(10)/19 = 2.632\text{ k}\Omega$$

$$\text{By inspection, } R_{Th} = R_3 + (R_1 + 875) \parallel (R_2 + 1803) = 4090\ \Omega$$

$$V_{Th} = (3.5)(R_2 + 1803) / (R_1 + 875 + R_2 + 1803) = 1.713\text{ V}$$

$$\text{Then, } P_{abs} = \left( \frac{V_{Th}}{R_{Th} + R} \right)^2 \frac{1}{R}$$

(a) 175 nW

(b) 1.65 nW

(c) 24.38 pW

(d) 2.02 aW

67. (a) Change the 25 V source to 10 V; then the two legs are identical.

(b) A load resistance equal to  $R_{Th}$  will absorb maximum power. In this case, shorting the two voltage sources and looking into the open terminals yields

$$R_{Th} = 15 \parallel (10 + 5) = 7.5 \Omega$$

68. As constructed, we may find the power delivered to the  $2.57 \Omega$  resistor by first determining the Thévenin equivalent. Removing the load resistor, we define a clockwise current  $i$  such that

$$5i - 25 + 10i + 15i + 10 = 0 \quad \text{or} \quad i = 0.5 \text{ A}$$

$$\text{Hence, } V_{\text{Th}} = V_{\text{oc}} = 15i + 10 = 17.5 \text{ V.}$$

$$R_{\text{Th}} = 15 \parallel (5 + 10) = 7.5 \Omega$$

$$\text{Hence, the power dissipated by the } 2.57 \Omega \text{ resistor is } \left( 17.5 \frac{2.57}{2.57 + 7.5} \right)^2 \left( \frac{1}{2.57} \right) = 7.76 \text{ W}$$

We need twice this, or 15.52 W so the voltage needed across the resistor must increase by a factor of  $\sqrt{2}$ . We can achieve this by increasing  $V_{\text{Th}}$  by the same factor. This is accomplished by simply scaling both voltage sources by the same factor.

Hence, 25 V is replaced with 35.4 V and 10 V is replaced with 14.1 V.

69. (a)  $R_{Th} = 5 \parallel [1.8 + 5.4 + 3] = 3.355 \Omega$

(b) Retain  $R_{Th} = 3.355 \Omega$  so leave all resistors unchanged.

Power to load is three times too large, so the voltage across the load is  $\sqrt{3}$  times too

large. Thus, reduce all sources by  $\frac{1}{\sqrt{3}}$ :

1.2 A becomes 692.8 mA

0.8 A becomes 461.9 mA

0.1 A becomes 57.74 mA

70. We first simplify the circuit and obtain its Thévenin equivalent.

Choose the bottom node as the reference. Designate the top left nodal voltage  $V_1$  and the top right nodal voltage  $V_2$ . Then

$$\frac{V_1}{8.4} + \frac{V_1 - V_2}{1.8} = -0.4$$

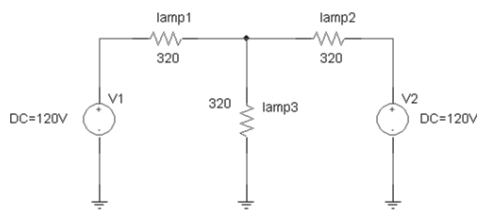
$$\frac{V_2}{5} + \frac{V_2 - V_1}{1.8} = 0.1$$

Solving,  $V_2 = V_{Th} = -769.7 \text{ mV}$

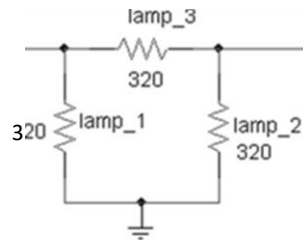
By inspection  $R_{Th} = 5 \parallel (8.4 + 1.8) = 3.355 \Omega$

To precisely mimic the behavior of the circuit at the open terminals, the battery should have an open circuit voltage of 769.7 mV, and an internal series resistance of 3.355  $\Omega$ . We note that due to the sign of the voltage, the '+' terminal of the battery should be connected in place of the bottom terminal of the load.

71. <Design> To solve this problem, we need to assume that “45 W” is a designation that applies when 120 Vac is applied directly to a particular lamp. This corresponds to a current draw of 375 mA, or a light bulb resistance of  $120 / 0.375 = 320 \Omega$ .



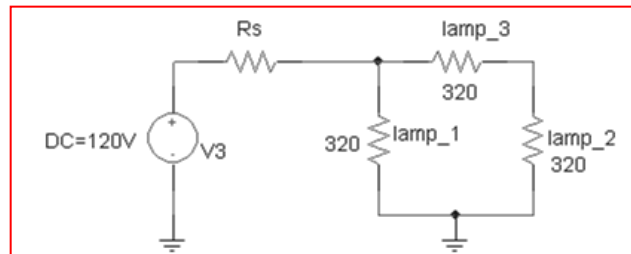
Original wiring scheme



New wiring scheme

In the original wiring scheme, Lamps 1 & 2 draw  $(40)^2 / 320 = 5 \text{ W}$  of power each, and Lamp 3 draws  $(80)^2 / 320 = 20 \text{ W}$  of power. Therefore, none of the lamps is running at its maximum rating of 45 W. We require a circuit which will deliver the same intensity after the lamps are reconnected in a  $\Delta$  configuration. Thus, we need a total of 30 W from the new network of lamps.

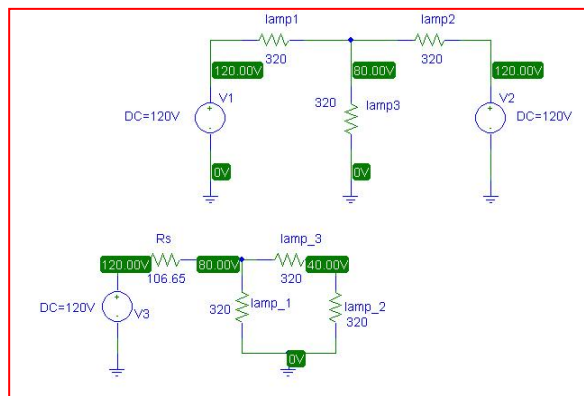
There are several ways to accomplish this, but the simplest may be to just use one 120-Vac source connected to the left port in series with a resistor whose value is chosen to obtain 30 W delivered to the three lamps.



In other words,

$$\left[ \frac{120 \cdot 213.3}{Rs + 213.3} \right]^2 \frac{1}{320} + 2 \left[ \frac{60 \cdot 213.3}{Rs + 213.3} \right]^2 \frac{1}{320} = 30$$

Solving, we find that we require  $Rs = 106.65 \Omega$ , as confirmed by the PSpice simulation below, which shows that both wiring configurations lead to one lamp with 80-V across it, and two lamps with 40 V across each.



72. (a) Source transformation can be used to simplify either nodal or mesh analysis by having all sources of one type. Otherwise, repeated source transformations can in many instances be used to reduce the total number of components, provided none of the elements involved are of interest.
- (b) For example, if the transformations involve an element whose voltage or current is of interest, since that information will be lost.
- (c) We do, indirectly, as their controlling variables will be scaled.
- (d) This is the same as replacing the source with a short circuit, so theoretically any current value is possible.
- (e) This is the same as replacing the source with an open circuit, so theoretically any voltage is possible.

73. (a) Define a nodal voltage  $V_1$  at the top of the current source  $I_S$ , and a nodal voltage  $V_2$  at the top of the load resistor  $R_L$ . Since the load resistor can safely dissipate 1 W, and we know that

$$P_{R_L} = \frac{V_2^2}{1000}$$

then  $V_2|_{\max} = 31.62\text{V}$ . This corresponds to a load resistor (and hence lamp) current of 32.62 mA, so we may treat the lamp as a  $10.6\Omega$  resistor.

Proceeding with nodal analysis, we may write:

$$I_S = V_1/200 + (V_1 - 5V_x)/200 \quad [1]$$

$$0 = V_2/1000 + (V_2 - 5V_x)/10.6 \quad [2]$$

$$V_x = V_1 - 5V_x \text{ or } V_x = V_1/6 \quad [3]$$

Substituting Eq. [3] into Eqs. [1] and [2], we find that

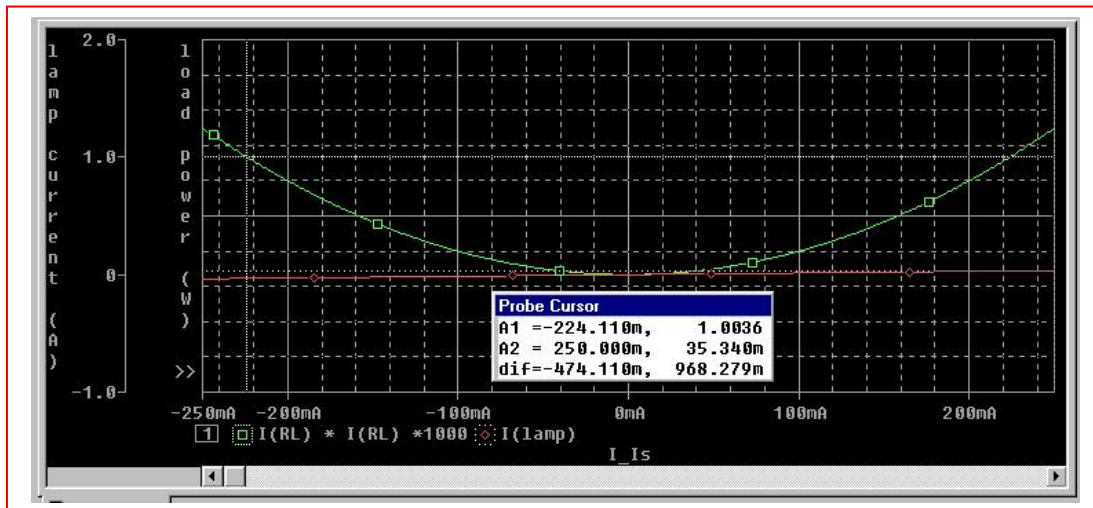
$$7V_1 = 1200I_S \quad [1]$$

$$-5000V_1 + 6063.6V_2 = 0 \quad [2]$$

Substituting  $V_2|_{\max} = 31.62\text{V}$  into Eq. [2] then yields  $V_1 = 38.35\text{V}$ , so that

$$I_S|_{\max} = (7)(38.35)/1200 = 223.7\text{ mA.}$$

- (b) Verification.



The lamp current does not exceed 36 mA in the range of operation allowed (*i.e.* a load power of  $< 1\text{ W}$ .) The simulation result shows that the load will dissipate slightly more than 1 W for a source current magnitude of 224 mA, as predicted by hand analysis.

74. <Design> One possible solution:

$$I_{\max} = 35 \text{ mA}$$

$$R_{\min} = 47 \Omega$$

$$R_{\max} = 117 \Omega$$

With only 9 V batteries and standard resistance values available, we begin by neglecting the series resistance of the battery.

We choose a single 9 V battery in series with a resistor R and the LED.

$$\text{Then, } I = 9/(R + R_{\text{LED}}) < 35 \text{ mA}$$

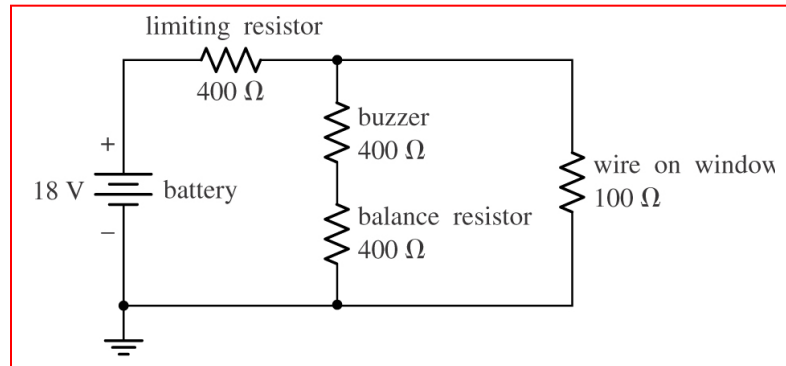
$$\text{Or } R + R_{\text{LED}} > 9/35 \times 10^{-3}$$

For safety, we design assuming the minimum LED resistance and so must select

$$R > 9/35 \times 10^{-3} - 47 \text{ or } R > 210 \Omega$$

The closest standard resistance value is 220  $\Omega$ .

75. <Design> We note that the buzzer draws 15 mA at 6 V, so that it may be modeled as a 400  $\Omega$  resistor. One possible solution of many, then, is:



Note: Construct the 18-V source from 12 1.5 V batteries in series, and the two 400  $\Omega$  resistors can be fabricated by soldering 400 1  $\Omega$  resistors in series. When the 100  $\Omega$  resistor is present, it shunts enough current to reduce the voltage across the buzzer to less than the required minimum voltage of 6 V.

76. (a) We remove  $R_L$ , select the bottom node as the reference, and assign the top open terminal the nodal voltage  $V_{oc}$ . Then,

$$1 = \frac{V_{oc}}{5} + \frac{V_{oc}}{2} + \frac{V_{oc} - (-3)}{3}$$

Solving,  $V_{oc} = 0 = V_{Th}$ .

Looking into the open terminals,  $R_{Th} = 5 \parallel 2 \parallel 3 = 0.968 \Omega$ .

(b) We transform the 3 V source to a  $3/3 = 1$  A source, arrow pointing down. This is in parallel with the original 1 A source. The arrangement leads to a net current of zero. Hence, the Norton equivalent is simply  $R_{Th} = 0.968 \Omega$ .

(c) Since  $V_{Th} = I_N = 0$ , the power supplied to  $R_L$  will be zero in either case.