1.

(a) $$\begin{bmatrix} -4 & 2 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

Solving, $v_1 = -2.056$ and $v_2 = 0.389$

(b) $$\begin{bmatrix} -1 & 0 & 2 \\ 2 & 1 & -5 \\ 4 & 5 & 8 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 6 \end{bmatrix}$$

Solving, $v_1 = -8.667$ $v_2 = 8.667$ $v_3 = -0.3333$

2\.

(a) Grouping terms,

$$\left(\frac{1}{5}+\frac{1}{22}+\frac{1}{3}\right)v_1 + \left(-\frac{1}{22}\right)v_2 + \left(-\frac{1}{3}\right)v_3 = 3$$

$$\left(-\frac{1}{22}\right)v_1 + \left(\frac{1}{22}+\frac{1}{14}\right)v_2 + \left(-\frac{1}{14}\right)v_3 = 1$$

$$\left(-\frac{1}{3}\right)v_1 + \left(-\frac{1}{14}\right)v_2 + \left(\frac{1}{10}+\frac{1}{3}+\frac{1}{14}\right)v_3 = 0$$

Solving,

v_1 = 48.475 V
v_2 = 155.32 V
v_3 = 53.991 V

(b) In Matlab,

```
A=[1/5+1/22+1/3 -1/22 -1/3; -1/22 1/22 -1/14; -1/3 -1/14 1/10+1/3+1/14]
B=[3; 1; 0]

v=A\B

A =

    0.5788   -0.0455   -0.3333
   -0.0455    0.0455   -0.0714
   -0.3333   -0.0714    0.5048


B =

     3
     1
     0

v =

   48.4753
  155.3184
   53.9910
```

3.

(a) Grouping terms,

$$\left(\frac{1}{2}-\frac{1}{12}+\frac{1}{19}\right)v_1 + \left(-\frac{1}{12}\right)v_2 + \left(-\frac{1}{19}\right)v_3 = 7$$

$$\left(-\frac{1}{12}\right)v_1 + \left(\frac{1}{12}+\frac{1}{2}\right)v_2 + \left(-\frac{1}{2}\right)v_3 = 15$$

$$\left(-\frac{1}{19}\right)v_1 + \left(-\frac{1}{2}\right)v_2 + \left(\frac{1}{7}+\frac{1}{19}+\frac{1}{2}\right)v_3 = 4$$

Solving,

v_1 = 42.473 V
v_2 = 102.84 V
v_3 = 82.896 V

(b) In Matlab,

```
A=[1/2-1/12+1/19 -1/12 -1/19; -1/12 1/12+1/2 -1/2; -1/19 -1/2 …
1/7+1/19+1/2]
B=[7; 15; 4]

v=A\B

      A =

    0.4693   -0.0833   -0.0526
   -0.0833    0.5833   -0.5000
   -0.0526   -0.5000    0.6955


B =

     7
    15
     4


v =

   42.4732
  102.8356
   82.8960
```

4.

The corrected code is as follows (note there were no errors in the e2 equation):

```
>> e1 = '3 = v1/7 - (v2 - v1)/2 + (v1 - v3)/3';
>> e2 = '2 = (v2 - v1)/2 + (v2 - v3)/14';
>> e3 = '0 = v3/10 + (v3 - v1)/3 + (v3 - v2)/14';
>> a = solve(e1,e2,e3,'v1','v2','v3');
>> a.v1
ans =
1178/53
>> a.v2
ans =
9360/371
>> a.v3
ans =
6770/371
```

5.

Our nodal equations are:

$$5 = \frac{v_1}{1} + \frac{v_1 - v_2}{5} \qquad [1]$$

$$-4 = \frac{v_2}{2} + \frac{v_2 - v_1}{5} \qquad [2]$$

Solving, $v_1 = 3.375$ V and $v_2 = -4.75$ V.
Hence,

$$i = (v_1 - v_2)/5 = \boxed{1.625 \text{ A}}$$

6.

Define nodal voltages v_1 and v_2 on the top left and top right nodes, respectively; the bottom node is our reference node. Our nodal equations are then,

$$-3 = \frac{v_1}{3} + \frac{v_1 - v_2}{2} \Rightarrow (2+3)v_1 - 3v_2 = -18$$

$$2 = v_2 + \frac{v_2 - v_1}{2} \Rightarrow -v_1 + (2+1)v_2 = 4$$

Solving the set where terms have been grouped together,

v_1 = -3.5 V and v_2 = 166.7 mV

$P_{1\Omega} = (v_2)^2/1$ = **27.79 mW**

7.

Define nodal voltages v_1 and v_2 on the top left and top right nodes, respectively; the bottom node is our reference node.

Note the current $i_x = v_1/4$

Our nodal equations are then,

$$\frac{v_1 - v_2}{2} + \frac{v_1}{4} = 2$$

$$\frac{v_2 - v_1}{2} + \frac{v_2}{3} + 0.5\left(\frac{v_1}{4}\right) = 0$$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2}+\frac{1}{8} & \frac{1}{2}+\frac{1}{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$v_1 = 3.8095V$$

$$v_2 = 1.7143V$$

$$\boxed{i_x = \frac{v_1}{4} = 0.9524A}$$

8.

Our two nodal equations are:

$$2 = \frac{v_1}{9} + \frac{v_1 - v_2}{1} \Rightarrow 10v_1 - 9v_2 = 18$$

$$15 = \frac{v_2}{2} + \frac{v_2 - v_1}{1} \Rightarrow -2v_1 + 3v_2 = 30$$

Solving, $v_1 = 27$ V and $v_2 = 28$ V. Thus, $\boxed{v_1 - v_2 = -1 \text{ V}}$

9.

We have three nodes in the circuit. Let us assign the reference node at the bottom of the circuit. We can then assign the top left node as v_1 (top left) and the node at the top right is already defined as 2 V relative to the reference. We therefore only have one unknown node v_1. While the voltage source suggests that we may need to use the procedure for a supernode, it is not necessary since the voltage source is directly specifying a node connected to the reference terminal. Writing KCL at v_1 yields

$$\frac{v_1}{3} + \frac{v_1 - 2}{1} = 2$$

$$v_1\left(\frac{1}{3} + \frac{1}{1}\right) = 4$$

$$v_1 = 3V$$

The problem asks us to find v_x, which is $v_1 - 2 = 1$ V

$$\boxed{v_x = 1V}$$

10.

Let us define the reference as the bottom node of the circuit. Notice that we essentially have two 'parts' of the circuit. The left part of the circuit is a voltage divider, where we see that

$$v_x = 12\ V \frac{2k}{2k+4k} = 4\ V$$

This value is then used to define the dependent source. We can then perform nodal analysis by writing a KCL equation at node v_o

$$\frac{v_o}{50} = 0.1 - 0.015(4)$$

$$\boxed{v_o = 2\ V}$$

11. Define v_1 across the 10 A source, '+' reference at the top.
Define v_2 across the 2.5 A source, '+' reference at the top.
Define v_3 across the 200 Ω resistor, '+' reference at the top.

Our nodal equations are then

$$10 = \frac{v_1}{20} + \frac{v_1 - v_p}{40} \qquad [1]$$

$$0 = \frac{v_p - v_1}{40} + \frac{v_p - v_2}{50} + \frac{v_p}{100} \qquad [2]$$

$$2 - 2.5 = \frac{v_2 - v_p}{50} + \frac{v_2 - v_3}{10} \qquad [3]$$

$$5 - 2 = \frac{v_3 - v_2}{10} + \frac{v_3}{200} \qquad [4]$$

Solving, $v_p = 171.6$ V

12. Choose the bottom node as the reference node. Then, moving left to right, designate the following nodal voltages along the top nodes: v_1, v_2, and v_3, respectively.

Our nodal equations are then

$$8+4=\frac{v_1-v_3}{3}+\frac{v_1-v_2}{3} \qquad [1]$$

$$-4=\frac{v_2}{5}+\frac{v_2-v_1}{3}+\frac{v_2-v_3}{1} \qquad [2]$$

$$-5=\frac{v_3-v_1}{3}+\frac{v_3-v_2}{1}+\frac{v_3}{7} \qquad [3]$$

Solving, $v_1 = 26.73$ V, $v_2 = 8.833$ V, $v_3 = 8.633$ V

$v_{5\Omega} = v_2 =$ 8.833 V

Thus, $P_{7\Omega} = (v_3)^2/7 =$ 10.65 W

13. Assign the following nodal voltages: v_1 at top node; v_2 between the 1 Ω and 2 Ω resistors; v_3 between the 3 Ω and 5 Ω resistors, v_4 between the 4 Ω and 6 Ω resistors. The bottom node is the reference node.

Then, the nodal equations are:

$$2 = \frac{v_1 - v_2}{1} + \frac{v_1 - v_3}{3} + \frac{v_1 - v_4}{4} \qquad [1]$$

$$-3 = \frac{v_2}{2} + \frac{v_2 - v_1}{1} \qquad [2]$$

$$3 = \frac{v_3}{5} + \frac{v_3 - v_1}{3} + \frac{v_3 - v_4}{7} \qquad [3]$$

$$0 = \frac{v_4}{6} + \frac{v_4 - v_1}{4} + \frac{v_4 - v_3}{7} \qquad [4]$$

Solving,

$$v_1 = 3.3355V$$

$$v_2 = 0.2237V$$

$$v_3 = 6.7604V$$

$$v_4 = 3.2164V$$

$$i_3 = \frac{v_3}{5} \boxed{= 1.3521A}$$

14. First, we note that it is possible to separate this circuit into two parts, connected by a single wire (hence, the two sections cannot affect one another).

For the left-hand section, our nodal equations are:

$$2 = \frac{v_1}{2} + \frac{v_1 - v_3}{6}$$

$$-2 = \frac{v_2}{5} + \frac{v_2 - v_3}{2} + \frac{v_2 - v_4}{10}$$

$$1 = \frac{v_3 - v_1}{6} + \frac{v_3 - v_2}{2} + \frac{v_3 - v_4}{5}$$

$$0 = \frac{v_4 - v_2}{10} + \frac{v_4 - v_3}{5} + \frac{v_4}{5}$$

Solving,

v_1 = 3.078 V
v_2 = -2.349 V
v_3 = 0.3109 V
v_4 = -0.3454 V

For the right-hand section, our nodal equations are:

$$-2 = \frac{v_5}{1} + \frac{v_5 - v_7}{4}$$

$$2 = \frac{v_6}{4} + \frac{v_6 - v_8}{4} + \frac{v_6 - v_7}{2}$$

$$6 = \frac{v_7 - v_5}{4} + \frac{v_7 - v_6}{2} + \frac{v_7 - v_8}{10}$$

$$0 = \frac{v_8}{1} + \frac{v_8 - v_6}{4} + \frac{v_8 - v_7}{10}$$

Solving,

v_5 = 1.019 V
v_6 = 9.217 V
v_7 = 13.10 V
v_8 = 2.677 V

15.

The bottom node is designated as the reference node. Naming our nodal voltages from left to right along the top nodes then: v_A, v_B, and v_C, respectively.

Our nodal equations are then:

$$0.02v_1 = \frac{v_A - v_C}{5} + \frac{v_A - v_B}{3} \quad [1]$$

$$10 = \frac{v_B - v_A}{3} + \frac{v_B - v_C}{2} \quad [2]$$

$$0 = \frac{v_C - v_B}{2} + \frac{v_C - v_A}{5} + \frac{v_C}{7} \quad [3]$$

However, we only have three equations but there are four unknowns (due to the presence of the dependent source). We note that $v_1 = v_C - v_B$. Substituting this into Eq. [1] and solving yields:

v_A = 77.02 V v_B = 83.49 V v_C = 67.80 V

Finally, $i_2 = (v_C - v_A)/5$ = **–1.844 A**

16. Select the bottom node as the reference node. The top node is designated as v_1, and the center node at the top of the dependent source is designated as v_2.

Our nodal equations are:

$$1 = \frac{v_1 - v_2}{5} + \frac{v_1}{2} \qquad [1]$$

$$v_x = \frac{v_2}{3} + \frac{v_2 - v_1}{5} \qquad [2]$$

We have two equations in three unknowns, due to the presence of the dependent source. However, $v_x = -v_2$, which can be substituted into Eq. [2]. Solving,

v_1 = 1.484 V and v_2 = 0.1936 V

Thus, $i_1 = v_1/2$ = **742 mA**

17.

We have four nodes in the circuit. Let us define the bottom node as the reference. The 5 V source automatically defines the node voltage on the positive terminal of the source (connected to the 10 ohm resistor). We are then left with two unknown node voltages at the two terminals of the 5 ohm resistor. Let us define the nodes as v_1 (the positive terminal of the voltage labeled v_x) and v_2 (the negative terminal of the voltage labeled v_x).

$$\frac{v_1-5}{10}+\frac{v_1}{80}+\frac{v_1-v_2}{5}=0$$

$$\frac{v_2-v_1}{5}+\frac{v_2}{15}=-0.3$$

Rewriting ,

$$v_1\left(\frac{1}{10}+\frac{1}{80}+\frac{1}{5}\right)+v_2\left(-\frac{1}{5}\right)=0.5$$

$$v_1\left(-\frac{1}{5}\right)+v_2\left(\frac{1}{5}+\frac{1}{15}\right)=-0.3$$

Solving, v_1=1.6923 V and v_2=0.1442 V

The unknown $v_x = v_1 - v_2 =$ 1.5481 V

18.

We first create a supernode from nodes 2 and 3. Then our nodal equations are:

$$3-5=\frac{v_1-v_3}{1}+\frac{v_1-v_2}{5} \qquad [1]$$

$$5-8=\frac{v_2}{3}+\frac{v_2-v_1}{5}+\frac{v_3}{2}+\frac{v_3-v_1}{1} \qquad [2]$$

We also require a KVL equation that relates the two nodes involved in the supernode:

$$v_2-v_3=4 \qquad [3]$$

Solving, v_1 = –8.6 V, v_2 = –3.6 V and v_3 = –7.6 V

19.

We name the one remaining node v_2. We may then form a supernode from nodes 1 and 2, resulting in a single KCL equation:

$$3-5=\frac{v_1}{5}+\frac{v_2}{9}$$

and the requisite KVL equation relating the two nodes is $v_1 - v_2 = 9$

Solving these two equations yields $v_1 = -3.214$ V

(and $v_2 = -12.213\ V$)

20.

We define v_1 at the top left node; v_2 at the top right node; v_3 the top of the 1 Ω resistor; and v_4 at the top of the 2 Ω resistor. The remaining node is the reference node.

We may now form a supernode from nodes 1 and 3. The nodal equations are:

$$-2 = \frac{v_3}{1} + \frac{v_1 - v_2}{10} \qquad [1]$$

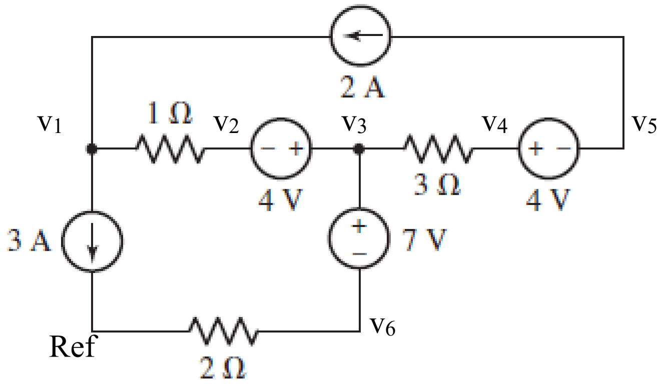
$$2 = \frac{v_4}{2} + \frac{v_4 - v_2}{4} \qquad [2]$$

By inspection, $v_2 = 5$ V and our necessary KVL equation for the supernode is $v_1 - v_3 = 6$. Solving,

$v_1 = 4.091$ V
$v_2 = 5$ V
$v_3 = -1.909$ V
$v_4 = 4.333$ V

21.

We first select a reference node then assign nodal voltages as follows:



There are two supernodes we can consider: the first is formed by combining nodes 2, 3 and 6. The second supernode is formed by combining nodes 4 and 5. However, since we are asked to only find the power dissipated by the 1 Ω resistor, we do not need to perform a complete analysis of this circuit.

At node 1, $-3 + 2 = (v_1 - v_2)/1$ or $v_1 - v_2 = -1$ V

Since this is the voltage across the resistor of interest, $P_{1\Omega} = (-1)^2/1 =$ **1 W**

22.

We begin by selecting the bottom center node as the reference node. Then, since 4 A flows through the bottom 2 Ω resistor, 4 V appears across that resistor.
Naming the remaining nodes (left to right) v_1, v_2, v_3, v_4, v_5, and v_6, respectively, we see two supernodes: combine nodes 2 and 3, and then nodes 5 and 6.

Our nodal equations are then

$$4-6=\frac{v_1-v_2}{14}$$

$$0=\frac{v_2-v_1}{14}+\frac{v_3-v_4}{7}$$

$$0=\frac{v_4-v_3}{7}+\frac{v_4-1}{2}+\frac{v_4-v_5}{7}$$

$$6=\frac{v_6}{3}+\frac{v_5-v_4}{7}$$

with

$$v_2-v_3=4$$

$$v_6-v_5=3$$

Solving, $v_4 = 0$. Thus, the current flowing out of the 1 V source is (1 – 0)/2 = 500 mA

and so the 1 V source supplies (1)(0.5) = **500 mW**

23.

We ***could*** do nodal analysis, but note that we can easily find the node voltages since they are connected by voltage sources!

$v_2 = 12$ V
$v_1 = 7$ V

24.

The 2 Ω resistor is shorted

$$\frac{v_1}{4}+\frac{v_1-v_2}{8}=2-0.5$$

$$v_2=12$$

$$v_1\left(\frac{1}{4}-\frac{1}{8}\right)=2-0.5+\frac{12}{8}$$

$$\boxed{v_1=24\ \text{V}}$$

25.

There is one large supernode connected by v_2 and v_1 (including 2 Ω resistor in parallel)

$$\frac{v_1}{4}+\frac{v_2}{10}=2+1$$

$$v_2-v_1=5$$

Solving,

$$v_1=7.1429\ V$$

$$v_2=12.143\ V$$

26.

A strong choice for the reference node is the bottom node, as this makes one of the quantities of interest (v_x) a nodal voltage. Naming the far left node v_1 and the far right node v_3, we are ready to write the nodal equations after making a supernode from nodes 1 and 3:

$$1+8=\frac{v_1-v_x}{8}+\frac{v_3}{2}$$

$$-8=\frac{v_x-v_1}{8}+\frac{v_x}{5}$$

Finally, our supernode's KVL equation: $v_3 - v_1 = 2v_x$

Solving, $v_1 = 31.76$ V and $v_x = -12.4$ V

Finally, $P_{supplied|1\ A} = (v_1)(1) = 31.76$ W

27.

We select the bottom center node as the reference. We next name the top left node v_1, the top middle node v_2, the top right node v_3, and the bottom left node v_4.

A supernode can be formed from nodes 1, 2 and 4. $v_3 = 4$ V by inspection.

Our nodal equation is then

$$-2 = \frac{v_4}{4} + \frac{v_2 - v_3}{2}$$

Then the KVL equation is $v_2 - v_4 = 0.5i_1 + 3$ where $i_1 = (v_2 - v_3)/2$.

Solving, $v_2 = 727.3$ mV and hence $i_1 = -1.636$ A

28.

Our nodal equations may be written directly, noting that two nodal voltages are available by inspection:

$$0 = \frac{v_x + 2}{1} + \frac{v_x}{1} + \frac{v_x - v_y}{4}$$

$$-1 = \frac{v_y - v_x}{4} + \frac{v_y - kv_y}{3}$$

Setting $v_x = 0$, Eq. [1] becomes $0 = 2 - v_y/4$ or $v_y = 8$ V.

Consequently, Eq. [2] becomes $-1 = 8/4 + (8 - 8k)/3$ or $k = 2.125$ (dimensionless)

29.

If we select the bottom node as our reference, and name the top three nodes (left to right) v_A, v_B and v_C, we may write the following nodal equations (noting that $v_B = 4v_1$):

$$2 = \frac{v_A - 4v_1}{2} + \frac{v_A - v_C}{3}$$

$$v_1 = \frac{v_C - 4v_1}{5} + \frac{v_C - v_A}{3}$$

And $v_1 = v_A - v_C$

Solving, $\boxed{v_1 = 480 \text{ mV}}$

30.

With the selected reference node, $v_1 = 1$ V by inspection. Proceeding with nodal analysis,

$$3 = \frac{v_2 - v_1}{1} + \frac{v_2 - v_3}{2}$$

$$-2v_x = \frac{v_3 - v_2}{2} + \frac{v_3 - v_4}{1}$$

$$0 = \frac{v_4}{3} + \frac{v_4 - v_3}{1} + \frac{v_4 - v_1}{4}$$

And to account for the additional variable introduced through the dependent source,

$v_x = v_3 - v_4$

Solving, $v_1 = 1$ V, $v_2 = 3.085$ V, $v_3 = 1.256$ V and $v_4 = 951.2$ mV

31.

Note that $v_3 = 9$ V.
The dependent voltage source at the top can be defined as a supernode between v_1 and v_2.

$$\frac{v_1 - 15}{4} - 2i_x + \frac{v_1 - 9}{4} + \frac{v_2 - 9}{6} + \frac{v_2 - v_4}{5} = 0$$

$$\frac{v_4 - v_2}{5} + \frac{v_4 - 9}{10} + \frac{v_4}{3} = 0$$

$$v_2 - v_1 = 5i_x$$

$$i_x = \frac{v_3 - v_4}{10} = \frac{9 - v_4}{10}$$

Substituting for i_x,

$$\frac{v_1 - 15}{4} - 2\left(\frac{9 - v_4}{10}\right) + \frac{v_1 - 9}{4} + \frac{v_2 - 9}{6} + \frac{v_2 - v_4}{5} = 0$$

$$\frac{v_4 - v_2}{5} + \frac{v_4 - 9}{10} + \frac{v_4}{3} = 0$$

$$v_2 - v_1 = 5\left(\frac{9 - v_4}{10}\right)$$

Grouping terms,

$$v_1\left(\frac{1}{4} + \frac{1}{4}\right) + v_2\left(\frac{1}{6} + \frac{1}{5}\right) + v_4\left(\frac{2}{10} - \frac{1}{5}\right) = \frac{15}{4} + 2\left(\frac{9}{10}\right) + \frac{9}{4} + \frac{9}{6}$$

$$v_2\left(-\frac{1}{5}\right) + v_4\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{3}\right) = \frac{9}{10}$$

$$v_1(-1) + v_2 + v_4\left(\frac{5}{10}\right) = 5\left(\frac{9}{10}\right)$$

Solving,

$$v_1 = 9.9184 \text{ V}$$
$$v_2 = 11.8386 \text{ V}$$
$$v_3 = 9 \text{ V}$$
$$v_4 = 5.1596 \text{ V}$$

32.

We define two clockwise flowing mesh currents i_1 and i_2 in the lefthand and righthand meshes, respectively. These are also the currents flowing out of the positive terminals of each voltage source.

Our mesh equations are then

$$4i_1 + 1(i_1 - i_2) = 1$$
$$1(i_2 - i_1) + 5(i_2) = 2$$

Solving,

$i_1 = 275.9$ mA and $i_2 = 379.3$ mA

33.

Our two mesh equations are:

$$5 + 7i_2 + 14(i_2 - i_1) = 0$$
$$14(i_1 - i_2) + 3i_1 - 12 = 0$$

Solving,

i_1 = 1.130 A and i_2 = 515.5 mA

34.

Our two mesh equations are:

$$9i_1 + 1(i_1 - i_2) = 15 + 11$$
$$1(i_2 - i_1) + 9i_2 = 21 - 11$$

Solving, $i_1 = 2.727$ A and $i_2 = 1.273$ A

35.

We need to construct three mesh equations:

$$-2 + (1)(i_1 - i_2) - 3 + 5(i_1 - i_3) = 0$$
$$(1)(i_2 - i_1) + 6i_2 + 9(i_2 - i_3) = 0$$
$$5(i_3 - i_1) + 3 + 9(i_3 - i_2) + 7i_3 = 0$$

Solving, i_1 = 989.2 mA, i_2 = 150.1 mA and i_3 = 157.0 mA

36.

We require three mesh equations:

$$-2 - 3 + 6i_1 - i_2 - 5i_3 = 0$$
$$0 = -i_1 + 16i_2 - 9i_3$$
$$3 - 5i_1 - 9i_2 + 21i_3 = 0$$

Solving, i_1 = 989.2 mA, i_2 = 150.2 mA, and i_3 = 157.0 mA

Thus,

$P_{1\Omega} = (i_2 - i_1)^2(1)$	=	703.9 mW
$P_{6\Omega} = (i_2)^2(6)$	=	135.4 mW
$P_{9\Omega} = (i_2 - i_3)^2(9)$	=	416.2 μW
$P_{7\Omega} = (i_3)^2(7)$	=	172.5 mW
$P_{5\Omega} = (i_3 - i_1)^2(5)$	=	3.463 W

37.

We could solve using mesh analysis:

There are four meshes, defining i_1 as the top mesh, and i_2, i_3, and i_4 from left to right on the bottom.

$$-5 + 12i_1 + 2(i_1 - i_3) = 0$$
$$-2 + 6i_2 + 4(i_2 - i_3) = 0$$
$$4(i_3 - i_2) + 2(i_3 - i_1) + 10 = 0$$
$$-10 + 8i_4 = 0$$

HOWEVER, we are simply asked to find the unknown voltage v_x. It is already defined on the circuit as 10 V! (This is intended as a lesson in carefully examining a circuit, rather than blindly employing a technique)

$v_x = 10$ V

38.

Define a clockwise mesh current i_y in the mesh containing the 10 A source. Then, define mesh currents i_1, i_2 and i_3, respectively, in the remaining meshes, starting on the left, and proceeding towards the right.

By inspection, $i_y = 10$ A
Then,

$$-3 + (8 + 4)i_1 - 4i_2 = 0$$
$$-4i_1 + (4 + 12 + 8)i_2 - 8i_3 = 0$$
$$-8i_2 + (8 + 20 + 5)i_3 - 20(10) = 0$$

Solving,

$$i_1 = 1.0458 \text{ A}$$
$$i_2 = 2.3874 \text{ A}$$
$$i_3 = 6.6394 \text{ A}$$

$i_x = i_3 - i_y$

$i_x = -3.3606$ A

39.

Define CW mesh currents i_1, i_2 and i_3 such that $i_3 - i_2 = i$.
Our mesh equations then are:

$0 = -2 + 8i_1 - 4i_2 - i_3$
$0 = 5i_2 - 4i_1$
$0 = 5i_3 - i_1$

Solving, $i_1 = 434.8$ mA, $i_2 = 347.8$ mA, and $i_3 = 86.96$ mA

Then, $i = i_3 - i_2 = \boxed{-260.8 \text{ mA}}$

40.

In the lefthand mesh, we define a clockwise mesh current and name it i_2.
Then, our mesh equations may be written as:

$4 - 2i_1 + (3 + 4)i_2 - 3i_1 = 0$
$-3i_2 + (3 + 5)i_1 + 1 = 0$

(note that since the dependent source is controlled by one of our mesh currents/variables/unknowns, these two equations suffice.)

Solving, $i_2 = -902.4$ mA so $P_{4\Omega} = (i_2)^2(4) = \boxed{3.257 \text{ W}}$

41.

(a) Define clockwise mesh currents i_1, i_2, i_3 and i_4. By inspection $i_1 = 4$ A and $i_4 = 1$ A. Define v_x across the dependent source with the bottom node as the reference node. Then,

$$2(i_2 - 4) + 1i_2 + v_x = 0$$

$$-v_x + 5i_3 + 2(i_3 - 1) = 0$$

Simplifying,

$$3i_2 = 8 - v_x$$

$$7i_3 = 2 + v_x$$

Adding these two equations cancels the v_x term

$$3i_2 + 7i_3 = 10$$

We note that $i_3 - i_2 = 5i_x$, where $i_x = i_4 - i_3 = 1 - i_3$.

$$-i_2 + 6i_3 = 5$$

Solving, $i_2 = 1$ A and $i_3 = 1$ A. Thus, $P_{1\Omega} = (1)(i_2)^2 =$ **1 W**

(b) Using nodal analysis, we define V_1 at the top of the 4 A source, V_2 at the top of the dependent source, and V_3 at the top of the 1 A source. The bottom node is our reference node.

Then,

$$4 = \frac{V_1}{2} + \frac{V_1 - V_2}{1}$$

$$5i_x = \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{5}$$

$$-1 = \frac{V_3}{2} + \frac{V_3 - V_2}{5}$$

and

$$i_x = -V_2/2$$

Solving, $V_1 = 6$ V and $V_2 = 5$ V

Hence, $P_{1\Omega} = (V_1 - V_2)^2/1 =$ **1 W**

42.

Define a clockwise mesh current i_1 for the mesh with the 2 V source; a clockwise mesh current i_2 for the mesh with the 5 V source, and clockwise mesh current i_3 for the remaining mesh.

Then, we may write

$-2 + (2 + 9 + 3)i_1 + 1 = 0$

which can be solved for $i_1 = 71.43$ mA

By inspection, $i_3 = -0.5v_x = -0.5(9i_1) =$ -321.4 mA

For the remaining mesh, $-1 + 10i_2 - 10i_3 - 5 = 0$ or $i_2 = 278.6$ mA

43.

We define four clockwise mesh currents. In the top left mesh, define i_1. In the top right mesh, define i_2. In the bottom left mesh, define i_3 (note that $i_3 = i_x$). In the last mesh, define i_4. Then, our mesh equations are:

$14i_1 - 7i_2 + 9 = 0$
$5i_3 - i_4 - 9 = 0$
$11i_2 + 0.2i_x = 0$
$5i_4 - 4i_2 - i_3 - 0.1v_a = 0$

where $v_a = -7i_1$.

Solving, $i_1 = -660.0$ mA, $i_2 = -34.34$ mA, $i_3 = 1.889$ A and $i_4 = 442.6$ mA.

Hence, $i_x = i_3 = 1.889$ A and $v_a = 462.0$ mV

44.

Our best approach here is to define a supermesh with meshes 1 and 3. Then,

$$-1 + 7i_1 - 7i_2 + 3i_3 - 3i_2 + 2i_3 = 0$$
$$-7i_1 + (7 + 1 + 3)i_2 - 3i_3 = 0$$
$$i_3 - i_1 = 2$$

Solving, $i_1 = -1.219$ A, $i_2 = -562.5$ mA and $i_3 = 781.3$ mA

45.

A supermesh from meshes 1 and 3 may be formed to simplify our analysis. Hence,

$-3 + 10i_1 - 11i_2 + 22i_3 = 0$
$-10i_1 + 16i_2 - i_3 = 0$
$-i_1 + i_3 = 5$

Solving, $i_1 = -4.238$ A, $i_2 = -2.601$ A and $\boxed{i_3 = 762.1 \text{ mA}}$

Hence, $P_{1\Omega} = (i_2 - i_3)^2(1) = \boxed{11.31 \text{ W}}$

46.

Define (left to right) three clockwise mesh currents i_2, i_3 and i_4. Then, we may create a supermesh from meshes 2 and 3. By inspection, $i_4 = 3$ A.

Our mesh/supermesh equations are:

$$7 + 5i_2 - 5i_1 + 11i_3 - 11i_1 + (1)i_3 - (1)i_4 + 5i_3 = 0$$
$$-5i_2 + (3 + 5 + 10 + 11)i_1 - 11i_3 = 0$$
$$i_3 - i_2 = 9$$

Solving, $i_1 = -874.3$ mA

$i_2 = -7.772$ A
$i_3 = 1.228$ A
$i_4 = 3$ A

Thus, the 1 Ω resistor dissipates $P_{1\Omega} = (1)(i_4 - i_3)^2 = 3.141$ W

47.

Note that current sources relate i_1, i_2, and i_3, defining one large supermesh. We can then define one supermesh equation, and two additional equations relating currents in the supermesh. Our three equations are

$$7 + 2200i_3 + 3500i_2 + 3100(i_2 - 2) + 8100(i_1 - 2) = 0$$
$$-i_1 + i_2 = 1$$
$$i_1 - i_3 = 3$$

Solving, $i_1 = 1.325$ A, $i_2 = 2.325$ A and $i_3 = -1.675$ A

48.

Defining mesh currents i_1 for the top left with the 12 ohm resistor, i_2 for the top right, i_3 for the middle bottom, and i_4 for the bottom right. Note that we already know the current for the bottom left mesh, $i = 2$ A. We have a supermesh connecting meshes 1 and 2, and will therefore have 3 mesh equations, and one equation relating the currents in the supermesh.

$$12i_1 + 5i_2 + 10(i_1 - i_3) + 5 = 0$$
$$8(i_3 - 2) - 5 + 10(i_3 - i_1) + 20(i_3 - i_4) = 0$$
$$20(i_4 - i_3) + 5 + 10i_4 = 0$$
$$i_2 - i_1 = 0.5$$

Solving,

$$i_1 = -14.7 \text{ mA}$$
$$i_2 = 485.3 \text{ mA}$$
$$i_3 = 710.2 \text{ mA}$$
$$i_4 = 306.8 \text{ mA}$$

$$\boxed{i_x = -i_4 = -306.8 \text{ mA}}$$

49.

By inspection, $i_1 = 5$ A.

$i_3 - i_1 = v_x/3$, and $v_x = 13i_3$. Hence, $i_3 = -1.5$ A.

In the remaining mesh,

$-13i_1 + 36i_2 - 11i_3 = 0$ so $i_2 = 1.347$ A.

50.

Define clockwise mesh current i_2 in the top mesh and a clockwise mesh current i_3 in the bottom mesh. Next, create a supermesh from meshes 2 and 3. Our mesh/supermesh equations are:

$-1 + (4 + 3 + 1)i_1 - 3i_2 - (1)i_3 = 0$
$(1)i_3 - (1)i_1 + 3i_2 - 3i_1 - 8 + 2i_3 = 0$
$i_2 - i_3 = 5i_1$

(Since the dependent source is controlled by a mesh current, there is no need for additional equations.)

Solving, $i_1 = 19$ A and hence $P_{supplied} = (1)i_1 =$ **19 W**

51.

Define clockwise mesh currents i_1, i_2 and i_3 so that $i_2 – i_3 = 1.8v_3$. Mesh 1 is on the left, mesh 2 is on the top right, mesh 3 is on the bottom right.

We form a supermesh from meshes 2 and 3 since they share a (dependent) current source.

Our supermesh/mesh equations are then

$–3 + 7i_1 – 4i_2 – 2i_3 = 0$
$–5 + i_3 + 2(i_3 – i_1) + 4(i_2 – i_1) = 0$
Also,
$i_2 – i_3 = 1.8v_3$ where $v_3 = i_3(1) = i_3$. Hence, $i_2 – i_3 = 1.8i_3$

Solving above, $i_1 = –2.138$ A, $i_2 = –1.543$ A and $i_3 = 551.2$ mA

$$\boxed{v_3 = 551.2 \text{ mV}}$$

52.

With the top node naturally associated with a clockwise mesh current i_a, we name (left to right) the remaining mesh currents (all defined flowing clockwise) as i_1, i_2 and i_3, respectively.

We create a supermesh from meshes 'a' and '2', noting that $i_3 = 6$ A by inspection.

Then,

$$-4 + 3i_1 - 2i_a - 3i_a = 0$$
$$2i_a + 3i_a - 3i_1 + 10i_a + 4i_a - 4(6) + 5i_2 - 5(6) = 0$$

Also, $i_2 - i_a = 5$

Solving, $i_a = 1.737$ A. Thus, $P_{10\Omega} = 10(i_a)^2 = \boxed{30.17\text{ W}}$

53.

(a) 4;

(b) Technically 5, but 1 mesh current is available "by inspection" so really just 4. We also note that a supermesh is indicated so the actual number of "mesh" equations is only 3.

(c) With nodal analysis we obtain i_5 by Ohm's law and 4 simultaneous equations. With mesh/supermesh, we solve 4 simultaneous equations and perform a subtraction. The difference here is not significant.

In the case of $v_{7\Omega}$, we could define the common node to the 3 A source and 7 W resistor as the refernce and obtain the answer with no further arithmetic steps. Still, we are faced with 4 simultaneous equations with nodal analysis so mesh analysis is still preferable in this case.

54. (a) Without employing the supernode technique, 4 nodal equations would be required. With supernode, only 3 nodal equations are needed plus a simple KVL equation. (Then simple division is necessary to obtain i_5).

(b) Although there are 5 meshes, one mesh current is available by inspection, so really only 4 mesh equations are required.

(c) The supernode technique is preferable here regardless; it requires fewer simultaneous equations.

55. (a) Nodal analysis requires 2 nodal equations and 2 simple subtractions
Mesh analysis requires 2 mesh equations and 2 simple multiplications

Neglecting the issues associated with fractions and grouping terms, neither appears to have a distinct advantage.

(b) Nodal analysis: we would form a supernode so 2 nodal equations plus one KVL equation. v_1 is available by inspection, v_2 obtained by subtraction.

Mesh analysis: 1 mesh equation, v_1 available by Ohm's law, v_2 by multiplication.

Mesh analysis has a slight edge here as no simultaneous equations required.

56. (a) Mesh analysis: Define two clockwise mesh currents i_1 and i_2 in the left and right meshes, respectively. A supermesh exists here.

Then, $2i_1 + 22 + 9i_2 = 0$
and $-i_1 + i_2 = 11$

Solving, $i_1 = 11$ A and $i_2 = 0$. Hence, $v_x = 0$.

(b) Nodal analysis: Define the top left node as v_1, the top right node as v_x.

We form a supernode from nodes 1 and x. Then,

$11 = v_1/2 + v_x/9$
and $v_1 - v_x = 22$

Solving, $v_1 = 22$ V and $v_x = 0$

(c) In terms of simultaneous equations, there is no real difference between the two approaches. Mesh analysis did require multiplication (Ohm's law) so nodal analysis had a very slight edge here.

57. (a) Nodal analysis: 1 supernode equation, 1 simple KVL equation. v_1 is a nodal voltage so no further arithmetic required.

(b) Mesh analysis: 4 mesh equations but two mesh currents available "by inspection" so only two mesh equations actually required. Then, invoking Ohm's law is required to obtain v_1.

(c) Nodal analysis is the winner, but it has only a slight advantage (no final arithmetic step). The choice of reference node will not change this.

58. (a) Using nodal analysis, we have 4 nodal voltages to obtain although one is available by inspection. Thus, 3 simultaneous equations are required to obtain v_1, from which we may calculate $P_{40\Omega}$.

(b) Employing mesh analysis, the existence of four meshes implies the need for 4 simultaneous equations. However, 2 mesh currents are available by inspection, hence only 2 simultaneous equations are needed. Since the dependent source relies on v_1, simple subtraction yields this voltage ($0.1\ v_1 = 6 – 4 = 2$ A).

Thus, v_1 can be obtained with NO simultaneous equations.

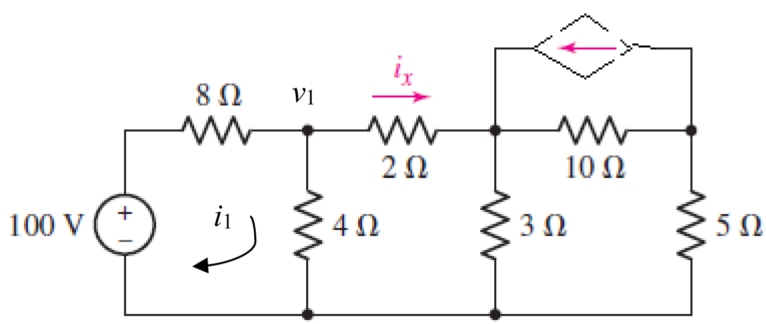
Mesh wins.

59. (a) Nodal analysis: 2 nodal equations, plus 1 equation for each dependent source that is not controlled by a nodal voltage = 2 + 3 = 5 equations.

(b) Mesh analysis: 3 mesh equations, 1 KCL equation, 1 equation for each dependent source not controlled by a mesh current = 3 + 1 + 2 = 6 equations.

60. <Design> One possible solution:

Replace the independent current source of Fig. 4.28 with a dependent current source.



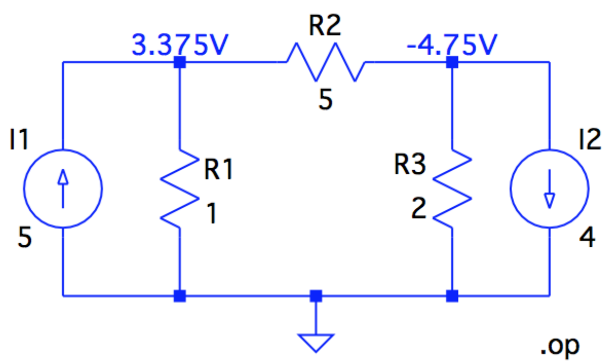
(a) Make the controlling quantity $8v_1$, i.e. depends on a nodal voltage.

(b) Make the controlling quantity $8i_1$, i.e. depends on a mesh current.

61.

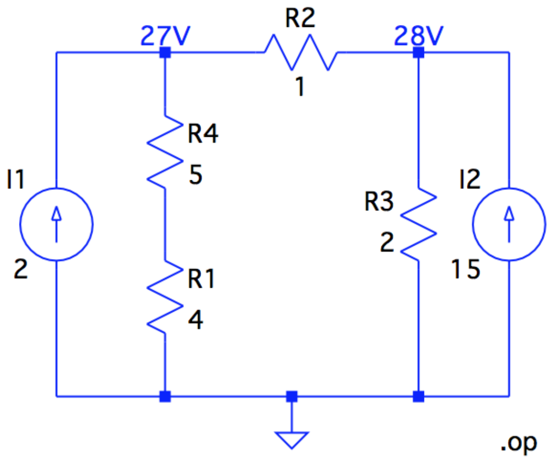
From SPICE, we get the node voltages
$v_1 = 3.375$ V and $v_2 = -4.75$ V.
Hence,

$$i = (v_1 - v_2)/5 = \boxed{1.625 \text{ A}}$$



62.

From SPICE, $v_1 = 27$ V and $v_2 = 28$ V. Thus, $v_1 - v_2 = -1$ V

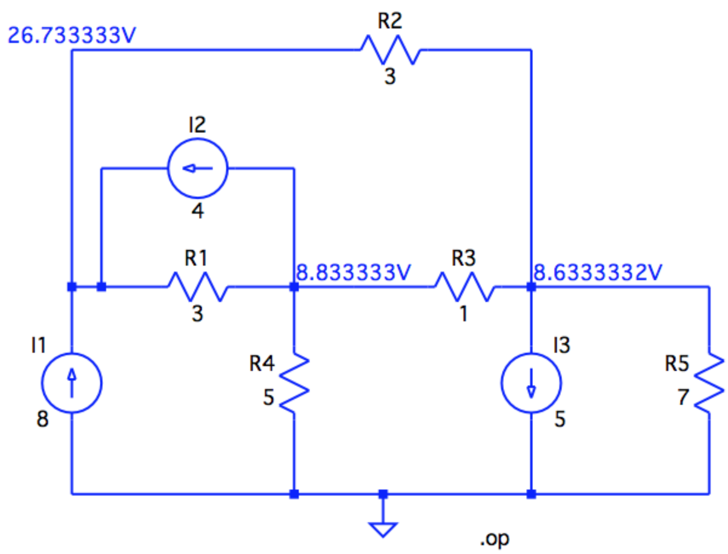


63.

From SPICE,

$$v_1 = 26.73 \text{ V}, \quad v_2 = 8.833 \text{ V}, \quad v_3 = 8.633 \text{ V}$$

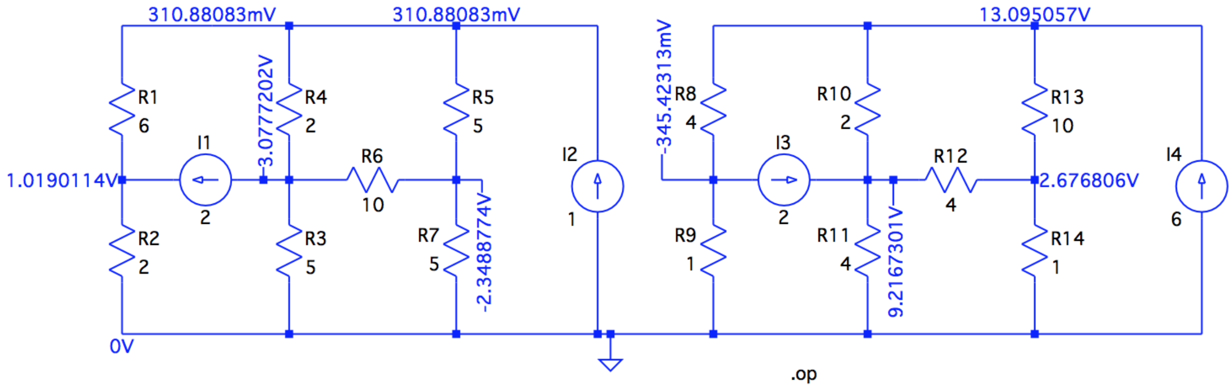
$v_{5\Omega} = v_2 = \boxed{8.833 \text{ V}}$



64.

Resulting node voltages are

v_1 = 3.078 V, v_2 = –2.349 V, v_3 = 0.3109 V, v_4 = -0.3454 V
v_5 = 1.019 V, v_6 = 9.217 V, v_7 = 13.10 V, v_8 = 2.677 V



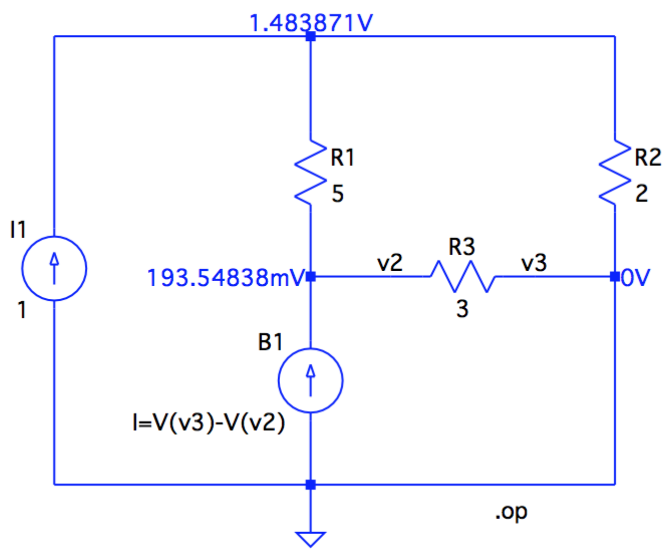
65.

From SPICE,

v_1 = 1.484 V and v_2 = 0.1936 V

Thus, $i_1 = v_1/2$ = 742 mA

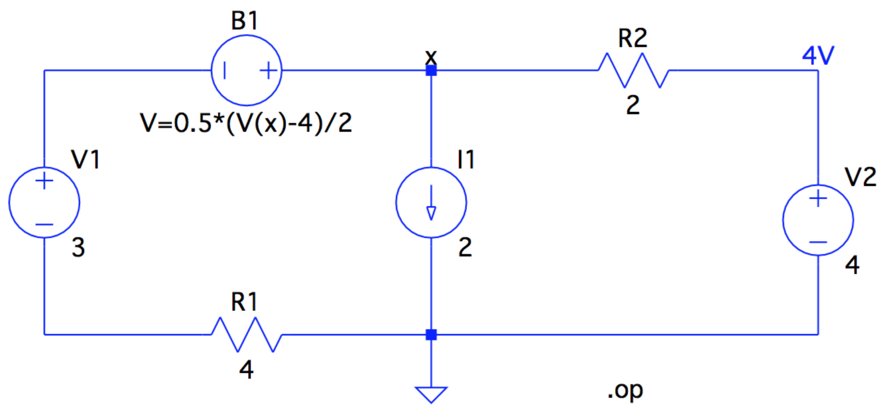
$v_x = -v_2 =$ −0.1936 V



66.

The schematic is almost correct, but has an error in the dependent source. The dependent source is determined by i_1, which is the current through the 2 ohm resistor. The current i2 should be the voltage at node x ***minus 4V***.

Revise the function of the dependent source to V=0.5*(V(x)-4)/2



The resulting current through R2 from the log file is the correct value of 1.63636 A

67.

(a) SPICE deck below

```
V1 1 0 DC 40
R1 1 2 11
R2 0 2 10
R3 2 3 4
R4 2 4 3
R5 0 3 5
R6 3 4 6
R7 3 9 2
R8 4 9 8
R9 0 9 9
R10 0 4 7
    .op
    .end
```

Output from log file:

```
Direct Newton iteration for .op point succeeded.
Operating Bias Point Solution:
      V(1)          40   voltage
      V(2)     8.34406   voltage
      V(3)     4.36699   voltage
      V(4)     5.19663   voltage
      V(9)      3.8487   voltage
```

(b) Hand calculations: nodal analysis is the only option.

$$0 = \frac{v_2 - 40}{11} + \frac{v_2}{10} + \frac{v_2 - v_3}{4} + \frac{v_2 - v_4}{3}$$

$$0 = \frac{v_3 - v_2}{4} + \frac{v_3}{5} + \frac{v_3 - v_4}{6} + \frac{v_3 - v_9}{2}$$

$$0 = \frac{v_4 - v_3}{6} + \frac{v_4}{7} + \frac{v_4 - v_9}{8} + \frac{v_4 - v_2}{3}$$

$$0 = \frac{v_9 - v_4}{8} + \frac{v_9 - v_3}{2} + \frac{v_9}{9}$$

Solving,

$v_1 = 40$ V (by inspection); $v_2 = 8.344$ V; $v_3 = 4.367$ V; $v_4 = 5.200$ V; $v_9 = 3.849$ V

68.

(a) The bulbs must be connected in parallel, or they would *all* be unlit.

(b) Parallel-connected means each bulb runs on 12 V dc. A power rating of 10 mW then indicates each bulb has resistance $(12)^2/(10\times10^{-3})$ = 14.4 kΩ

Given the high resistance of each bulb, the resistance of the wire connecting them is negligible.

SPICE Deck:

```
V1 1 0 DC 12
R1 1 0 327.3
.op
.end
```

SPICE Output:

```
Direct Newton iteration for .op point succeeded.
Operating Bias Point Solution:
    V(1)          12   voltage
    I(R1)   0.0366636   device_current
    I(V1)  -0.0366636   device_current
```

Total power = IV = (0.03666)(12) = **0.440 W**

(c) The equivalent resistance of 44 parallel-connected lights is

$$R_{eq} = \frac{1}{\left[(44)\left(\frac{1}{14400}\right)\right]} = 327.3\ \Omega$$. This would draw $(12)^2/327.3$ = **440 mW.**

69.

<Design> One possible solution:
We select nodal analysis, with the bottom node as the reference terminal. We then assign nodal voltages v_1, v_2, v_3 and v_4 respectively to the top nodes, beginning at the left and proceeding to the right.

We need $v_2 - v_3 = (2)(0.2) = 400$ mV
Arbitrarily select $v_2 = 1.4$ V, $v_3 = 1$ V.

So, element B must be a 1.4 V voltage source, and element C must be a 1 V voltage source. Choose A = 1 A current source, D = 1 A current source; F = 500 mV voltage source, E = 500 mV voltage source.

70.

(a) If the voltage source lies between any node and the reference node, that nodal voltage is readily apparent simply by inspection.

(b) If the current source lies on the periphery of a mesh, i.e. is not shared by two meshes, then that mesh current is readily apparent simply by inspection.

(c) Nodal analysis is based upon conservation of charge.

(d) Mesh analysis is based upon conservation of energy.

71.

(a) Although mesh analysis yields i_2 directly, it requires three mesh equations to be solved. Therefore, nodal analysis has a slight edge here since the supernode technique can be invoked.

We choose the node at the "+" terminal of the 30 V source as our reference. We assign nodal voltage v_A to the top of the 80 V source, and v_C to the "-" terminal of that source. v_B, the nodal voltage at the remaining node (the "-" terminal of the 30 V source), is seen by inspection to be -30 V (v_B = -30 [1]). Our nodal equations are then

$$0 = \frac{v_A - v_B}{10} + \frac{v_C}{30} + \frac{v_C}{40} \quad [2]$$

and $v_A - v_C = 80$ [3]

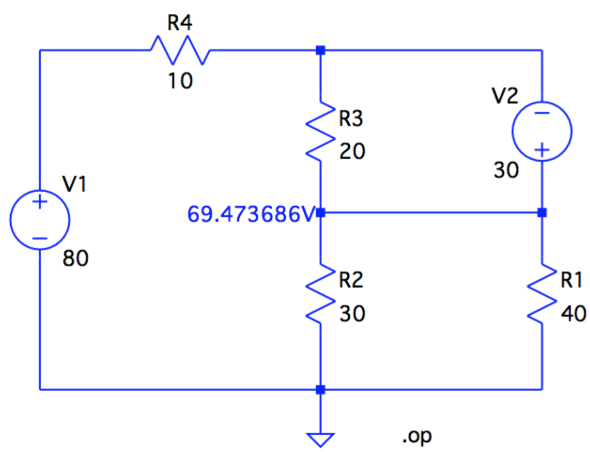
Solving,

$v_A = 10.53$ V, $v_B = -30$ V, and $v_C = -69.47$ V.

Hence

$$i_2 = -v_C/30 = \boxed{2.316 \text{ A}}$$

(b) SPICE schematic shown below, confirming the current i_2 (either from 69.47/30, or reading current from the log file). Note the difference in the voltage reference chosen in the schematic below – of course this does not change the result!



72.

(a) Define mesh currents i_1, i_2, and i_3, and write KVL equations:

$$-5 + 50i_1 + 100(i_1 - i_2) + 1.8 = 0$$

$$-1.8 + 100(i_2 - i_1) + 10i_2 + 100(i_2 - i_3) + 2.2 = 0$$

$$-2.2 + 100(i_3 - i_2) + 10i_3 + 100i_3 + 3.2 = 0$$

Rearranging terms

$$150i_1 - 100i_2 = 3.2$$

$$-100i_1 + 210i_2 - 100i_3 = -0.4$$

$$-100i_2 + 210i_3 = -1$$

Solving,

$$i_1 = 3.009 \times 10^{-2}\ A = 30.09\ mA$$

$$i_2 = 1.313 \times 10^{-2}\ A = 13.13\ mA$$

$$i_3 = 1.493 \times 10^{-3}\ A = 1.493\ mA$$

Translating into I_{Red}, I_{Green}, and I_{Blue},

$$I_{Red} = i_1 - i_2 = 30.09 - 13.13 = \boxed{16.96\ mA}$$

$$I_{Green} = i_2 - i_3 = 13.13 - 1.493 = \boxed{11.64\ mA}$$

$$I_{Blue} = i_3 = \boxed{1.493\ mA}$$

The differing current values for the LEDs means that you will get much more light from the red LED, and much less light from the Green LED.

(b) For the case of matching current levels, we know the current values given by the following.

$$I_{Red} = I_{Green} = I_{Blue} = 4\ mA$$

$$i_1 - i_2 = 4\ mA$$

$$i_2 - i_3 = 4\ mA$$

$$i_3 = 4\ mA$$

$$i_2 = i_3 + 4\ mA = 8\ mA$$

$$i_1 = i_2 + 4\ mA = 12\ mA$$

Revising our KVL equations, but now with known current values and unknown resistor values:

$$-5 + 50(12\ m) + R_1(4\ m) + 1.8 = 0$$

$$R_1 = \frac{5 - 1.8 - 50(12\ m)}{4\ m} = \boxed{650\ \Omega}$$

$$-1.8+650(-4\ m)+10(8\ m)+R_2(4\ m)+2.2=0$$

$$R_2=\frac{1.8-2.2+650(4\ m)-10(8\ m)}{4\ m}=\boxed{530\ \Omega}$$

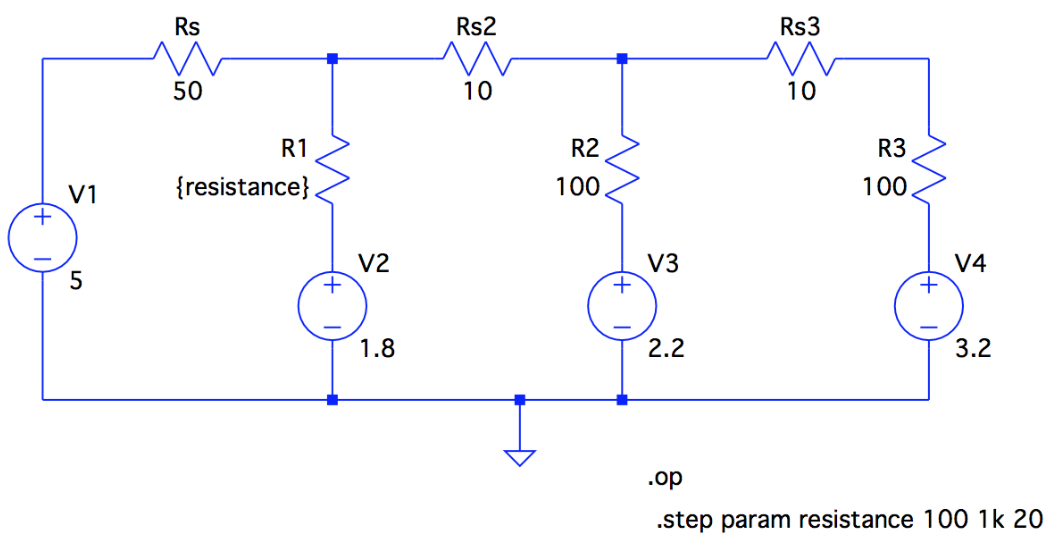
$$-2.2+530(-4\ m)+10(4\ m)+R_3(4\ m)+3.2=0$$

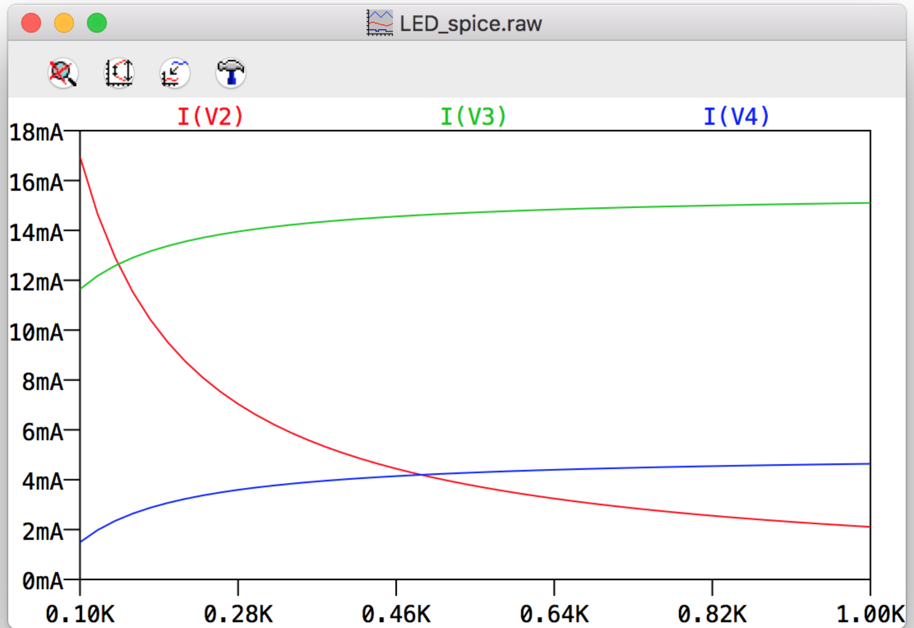
$$R_3=\frac{2.2-3.2+530(4\ m)-10(4\ m)}{4\ m}=\boxed{270\ \Omega}$$

73.

(a) Plot the current of all three LEDs as a function of R_1 and explain the result.

The schematic and resulting plot is shown below. The plot of current through V2 represents the red LED, the current through V3 represents the green LED, and the current through V4 represents the blue LED. For a low value of R1, the red LED will provide a low resistance path for current flow, and the red LED will be bright. As the resistance R1 is increased, the current through the red LED will quickly decrease, while the current through the green and blue LED will increase due to a decrease in resistance in those branches of the circuit.





(b) Find an RGB color chart and describe how the color changes with R_1 increasing from 100 to 1 kΩ.

RGB is given in terms of numbers from 0 to 255. The light intensity of the LED is directly proportional to the electrical current. For our circuit, the range is approximately from zero to 17 mA. The RGB numbers could then be approximated by multiplying by the current in mA by 255/17

For select resistance values:

R1 (Ω)	Red (mA)	Green (mA)	Blue (mA)	R	G	B	Color
100	16.96	11.64	1.49	254	175	22	Orange
200	9.51	13.38	3.07	143	201	46	Fern Green
500	4.10	14.64	4.22	62	220	63	Shamrock Green
1k	2.11	15.10	4.64	32	227	70	Bright Green

The color starts as orange, and rapidly changes to green as resistance R1 is increased.

(c) Find a value of R1 that could be used to achieve a khaki color approximating RGB hex code C2BD23, RGB (194,189,35).

Corresponds to current levels of I_{red} = 12.9 mA; I_{green} = 12.6 mA; I_{blue} = 2.3 mA

This occurs approximately at R_1 = 140 Ω

74.

(a) Derive an algebraic expression for V_{out} in terms of R_S, R_1, R_2, R_{light}, and R_{pot}.

Begin with mesh analysis defining mesh current i_1 on the left, and i_2 on the right (in clockwise directions)

$$-V_S + i_1 R_S + R_1(i_1 - i_2) + R_2(i_1 - i_2) = 0$$

$$R_2(i_2 - i_1) + R_1(i_2 - i_1) + R_{light} i_2 + R_{pot} i_2 = 0$$

Using Matlab symbolic math to solve for i_1 and i_2

```
syms i1 i2 Rs R1 R2 Rlight Rpot Vs

[soli1,soli2]=solve(i1*(Rs+R1+R2)+i2*(-R1-R2)==Vs, ...
          i1*(-R2-R1)+i2*(R2+R1+Rlight+Rpot)==0)
```

Results in

```
soli1 =

(Vs*(R1 + R2 + Rlight + Rpot))/(R1*Rlight + R2*Rlight + R1*Rpot + R2*Rpot
+ R1*Rs + R2*Rs + Rlight*Rs + Rpot*Rs)


soli2 =

(Vs*(R1 + R2))/(R1*Rlight + R2*Rlight + R1*Rpot + R2*Rpot + R1*Rs + R2*Rs
+ Rlight*Rs + Rpot*Rs)
```

V_{out} can be described in terms of mesh currents by

$$V_{out} = i_2 R_{pot} + (i_2 - i_1) R_2$$

Evaluating in Matlab

```
Vout=soli2*Rpot+(soli2-soli1)*R2
```

Resulting in

```
Vout =

(Rpot*Vs*(R1 + R2))/(R1*Rlight + R2*Rlight + R1*Rpot + R2*Rpot + R1*Rs +
R2*Rs + Rlight*Rs + Rpot*Rs) - R2*((Vs*(R1 + R2 + Rlight +
Rpot))/(R1*Rlight + R2*Rlight + R1*Rpot + R2*Rpot + R1*Rs + R2*Rs +
Rlight*Rs + Rpot*Rs) - (Vs*(R1 + R2))/(R1*Rlight + R2*Rlight + R1*Rpot +
R2*Rpot + R1*Rs + R2*Rs + Rlight*Rs + Rpot*Rs))
```

(b) Using the numerical values given in the circuit, calculate the value of R_{pot} required to balance the circuit at 500 lux, where R_{light} = 200 Ω.

Using the above, we can now substitute numerical values given and solve for the balanced case where V_{out}=0.

```
Rs=10;
R1=198;
R2=204;
Rlight=200;
Vs=12;

Vout=subs(Vout)

[solRpot]=solve(Vout==0)
```

Resulting in

```
Vout =

(4824*Rpot)/(412*Rpot + 86420) + 984096/(412*Rpot + 86420) - (2448*(Rpot +
602))/(412*Rpot + 86420)

solRpot =

6800/33
```

R_{pot} = 206.06 Ω

(c) If the resistance of the photoresistor decreases by 2% for a light increase to 600 lux (and assuming the resistance change with light is linear), what would the light level be if you measure V_{out} =150 mV?

Evaluate V_{out} at 600 lux.

R_{pot}=206.06 Ω for the balanced condition at 500 lux

R_{light} = 0.98×200 = 196 Ω.

Evaluating V_{out} in Matlab with these values,

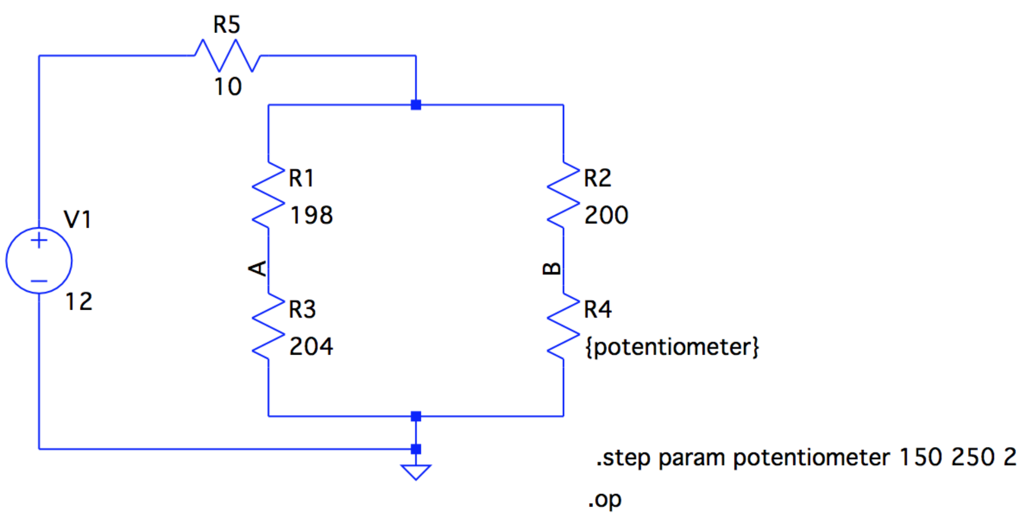
V_{out} = 57.704 mV

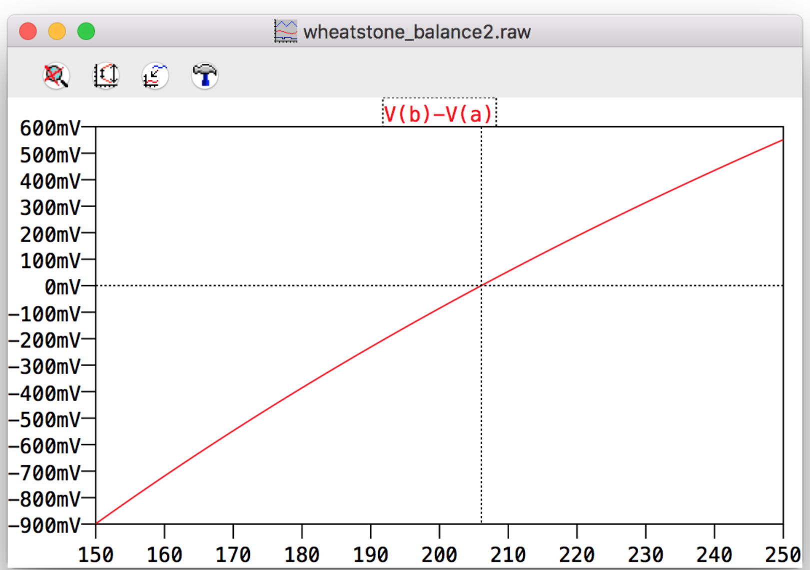
For a linear change,

$$Light = 500\ Lux + \frac{100\ Lux}{57.704\ mV} 150\ mV = 760\ Lux$$

75.

(a) Simulate the circuit for varying values of R_{pot} to balance the circuit at 500 lux, where R_{light} = 200 Ω. It is helpful to use a parameter sweep by defining a variable such as **{potentiometer}** (including the curly brackets) in the value for R_{pot} , and a SPICE directive such as **.step param potentiometer 150 250 2** to step the variable from 150 to 250 in steps of 2.



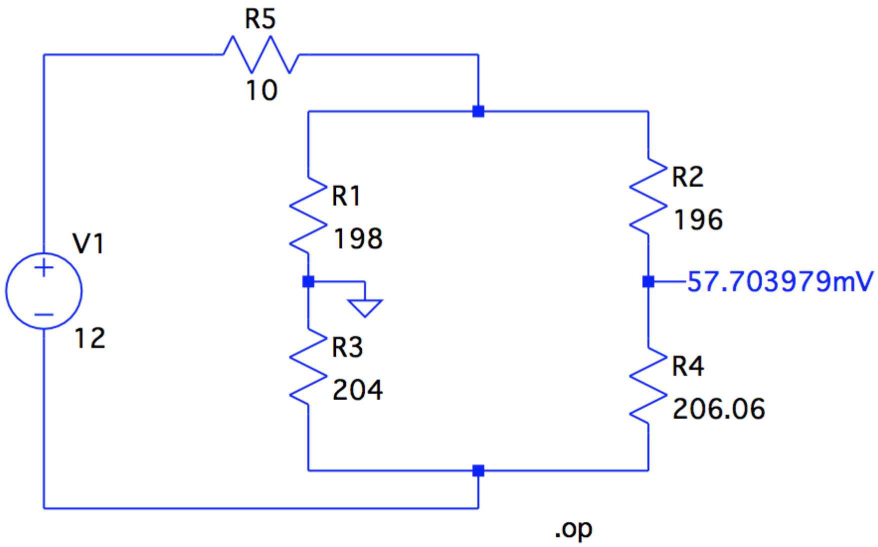


Balanced condition at $R_{pot} = 206.1\ \Omega$

(b) If the resistance of the photoresistor decreases by 2% for a light increase to 600 lux, use SPICE to find the resulting output voltage V_{out}.

R_{pot}=206.06 Ω for the balanced condition at 500 lux
$R_{light} = 0.98 \times 200 = 196\ \Omega$.

Simulating,



$V_{out} = 57.704$ mV